

Name: Solutions

Section: _____

Tutorial Section: _____

(If you don't remember your tutorial section, write the day and time)

WILFRID LAURIER UNIVERSITY

Waterloo, Ontario

Mathematics 129 – Introductory Calculus

Midterm – February 28, 2014

Instructor:

Section C – 11:30 am MWF – *S. Bauman*

Section D – 12:30 pm MWF – *S. Bauman*

Time Allowed: *80 minutes*

Total Value: *70 marks*

Number of Pages: *6 plus cover page*

Instructions:

Non-programmable, non-graphing calculators are permitted. No other aids are allowed.

Check that your test paper has no missing, blank, or illegible pages.

Answer in the spaces provided. Please note that questions are printed on both sides of the test pages.

Show all your work. Insufficient justification will result in a loss of marks.

Student Number: _____

[3 marks] 1. Solve the equation $\frac{1}{a-2} + \frac{1}{a+2} = \frac{6}{a^2-4}$.

multiply by a^2-4 ✓
 $a+2 + a-2 = 6$ ✓
 $2a = 6$
 $a = 3$ ✓

OR

$$\frac{a+2}{a^2-4} + \frac{a-2}{a^2-4} = \frac{6}{a^2-4}$$

$$\frac{2a}{a^2-4} = \frac{6}{a^2-4}$$

$$2a = 6$$

$$a = 3$$
 ✓

[4 marks] 2. Simplify the expression $\left(\frac{x^3}{2y}\right)^2 \div \frac{x^3y^2}{12}$. Write your answer using only positive exponents.

$$\frac{x^6}{4y^2} \cdot \frac{12}{x^3y^2} = \frac{3x^3}{y^4}$$

[5 marks] 3. Solve the inequality $x^4 - 2x^3 + x^2 > 0$. State your answer using interval notation.

$$x^4 - 2x^3 + x^2 = 0$$

$$x^2(x^2 - 2x + 1) = 0$$

$$x^2(x-1)^2 = 0$$

$$x = 0, 1$$

| | | | | | |
|--------------|---|---|---|---|---|
| | | 0 | 1 | | |
| x^2 | + | 0 | + | + | + |
| $(x-1)^2$ | + | + | + | 0 | + |
| $x^2(x-1)^2$ | + | 0 | + | 0 | + |

OR

let $f(x) = x^4 - 2x^3 + x^2$

| | | | |
|-------------------------|---|--------------------------|--|
| | 0 | 1 | |
| $f(-1) = 1 + 2 + 1 = 4$ | $f\left(\frac{1}{2}\right) = \frac{1}{16} - \frac{1}{4} + \frac{1}{4} = \frac{1}{16}$ | $f(2) = 16 - 16 + 4 = 4$ | |

| | | | | | |
|--------|---|---|---|---|---|
| $f(x)$ | + | 0 | + | 0 | + |
|--------|---|---|---|---|---|

$\therefore x \in (-\infty, 0) \cup (0, 1) \cup (1, \infty)$ ✓

[4 marks] 4. Solve the equation $\log(x^2 - 16) - \log(x + 4) = 1$. Give your answer(s) rounded to 2 decimal places.

$$\log\left(\frac{x^2 - 16}{x + 4}\right) = 1 \quad \checkmark$$

$$\log\left[\frac{(x-4)(x+4)}{x+4}\right] = 1 \quad \checkmark$$

$$\log(x-4) = 1 \quad \checkmark$$

$$10 = x - 4 \quad \checkmark$$

$$x = 14 \quad \checkmark$$

[7 marks] 5. Solve the system.

$$\begin{aligned} -x + 2y + z &= -1 \\ -3y - 7z &= -11 \\ 4x - 7y - 1z &= 9 \end{aligned}$$

$$\left[\begin{array}{ccc|c} -1 & 2 & 1 & -1 \\ 0 & -3 & -7 & -11 \\ 4 & -7 & -1 & 9 \end{array} \right] \xrightarrow{-R_1} \left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & -3 & -7 & -11 \\ 4 & -7 & -1 & 9 \end{array} \right] \xrightarrow{-4R_1 + R_3} \left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & -3 & -7 & -11 \\ 0 & 1 & 3 & 5 \end{array} \right] \quad \checkmark$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 3 & 5 \\ 0 & -3 & -7 & -11 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{array} \right] \xrightarrow{3R_2 + R_3} \left[\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array} \right] \quad \checkmark$$

$$\boxed{z = 2} \quad \checkmark$$

$$y + 6 = 5 \quad \boxed{y = -1} \quad \checkmark$$

$$x - 2(-1) - 1(2) = 1 \quad \boxed{x = 1} \quad \checkmark$$

[2 marks] 6. Complete the following tables of values so that the first table represents a function and the second table does not.

(a)

| x | f(x) |
|---|------|
| 1 | 2 |
| 2 | 3 |
| 3 | 4 |
| 4 | 5 |
| 3 | 4 |

must match \checkmark

(b)

| x | f(x) |
|---|------|
| 1 | 3 |
| 2 | 5 |
| 3 | 2 |
| 4 | 1 |
| 3 | 1 |

must be different \checkmark

- [4 marks] 7. Find the interest earned on \$2000 invested for 3 years at 2% annual interest compounded continuously.

$$A = Pe^{rt} \quad \checkmark$$

$$= 2000e^{0.02(3)} = 2123.67 \quad \checkmark$$

$$\text{Interest} = 2123.67 - 2000$$

$$= 123.67 \quad \checkmark$$

- [4 marks] 8. Are the lines represented by the equations $y = 5x + 4$ and $-10x + 2y + 3 = 0$ parallel, perpendicular or neither? Justify your answer.

$$y = 5x + 4 \Rightarrow \text{slope is } 5 \quad \checkmark$$

$$-10x + 2y + 3 = 0$$

$$2y = 10x - 3 \Rightarrow \text{slope is } 5 \quad \checkmark$$

$$y = 5x - \frac{3}{2}$$

The two lines are parallel \checkmark
since they have the same slope \checkmark

- [3 marks] 9. Using interval notation, give the domain of the function defined by $f(x) = \sqrt{\frac{2}{12x - 6}}$

$$12x - 6 > 0 \quad \checkmark$$

$$12x > 6$$

$$x > \frac{1}{2} \quad \checkmark$$

$$\text{domain} = \left(\frac{1}{2}, \infty\right) \quad \checkmark$$

- [2 marks] 10. Find the average rate of change of $f(w) = 3^{2w-3}$ over the interval $[2, 5]$.

$$\text{AROC} = \frac{f(5) - f(2)}{5 - 2} \quad \checkmark$$

$$= \frac{3^7 - 3}{3} = 728 \quad \checkmark$$

[3 marks] 11. Find the vertex of the parabola $y = 4x^2 - 16x + 13$.

$$y = 4(x^2 - 4x + 4) + 13 - 16 \quad \checkmark$$

$$y = 4(x-2)^2 - 3 \quad \checkmark$$

$$\text{vertex is } (2, -3) \quad \checkmark$$

OR

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right) \quad \checkmark$$

$$= \left(\frac{16}{2(4)}, f(2) \right)$$

$$= (2, -3) \quad \checkmark \quad \checkmark$$

[5 marks] 12. If $f(x) = 2\sqrt{x}$, using the definition of the derivative, find $f'(x)$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \checkmark$$

$$= \lim_{h \rightarrow 0} \frac{2\sqrt{x+h} - 2\sqrt{x}}{h} \cdot \frac{2\sqrt{x+h} + 2\sqrt{x}}{2\sqrt{x+h} + 2\sqrt{x}} \quad \checkmark$$

$$= \lim_{h \rightarrow 0} \frac{4(x+h) - 4x}{h(2\sqrt{x+h} + 2\sqrt{x})} \quad \checkmark$$

$$= \lim_{h \rightarrow 0} \frac{4h}{h(2\sqrt{x+h} + 2\sqrt{x})}$$

$$= \lim_{h \rightarrow 0} \frac{4}{2\sqrt{x+h} + 2\sqrt{x}} = \frac{4}{4\sqrt{x}} = \frac{1}{\sqrt{x}} \quad \checkmark \quad \checkmark$$

[4 marks] 13. Solve the equation $8^{x+2} = 684$. Give your answer(s) rounded to 2 decimal places.

$$\ln(8^{x+2}) = \ln 684 \quad \checkmark$$

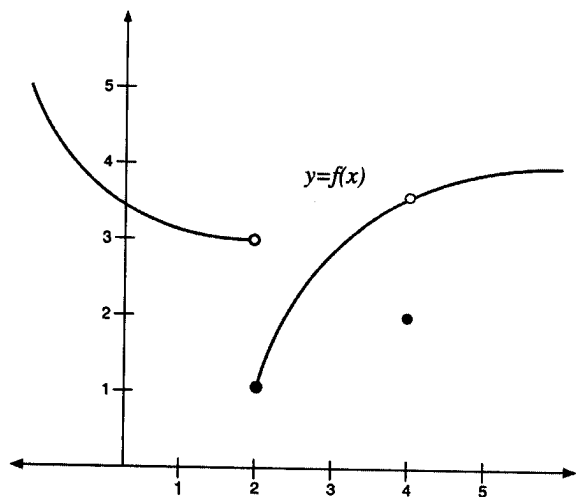
$$(x+2) \ln 8 = \ln 684 \quad \checkmark$$

$$x \ln 8 = \ln 684 - 2 \ln 8$$

$$x = \frac{\ln 684 - 2 \ln 8}{\ln 8} \quad \checkmark$$

$$x \approx 1.14 \quad \checkmark$$

- [3 marks] 14. Determine the values of (a) $\lim_{x \rightarrow 2^+} f(x)$ and (b) $\lim_{x \rightarrow 2} f(x)$, (c) $\lim_{x \rightarrow 4} f(x)$, if they exist, where $f(x)$ is represented in the figure below.



14.(a) Answer: 1

(b) Answer: DNE

(c) Answer: 3,6

[accept any answer from 3.5 to 3.8]

- [1 mark] 15. If $f(x) = x^3 + 2$ and $f(g(x)) = x^6 + 2$, what is $g(x)$?

$$\begin{aligned} f(g(x)) &= [g(x)]^3 + 2 = x^6 + 2 \\ [g(x)]^3 &= x^6 \\ g(x) &= x^2 \quad \checkmark \end{aligned}$$

- [2 marks] 16. Let $A = \begin{bmatrix} 1 & 3 \\ 6 & -1 \\ 2 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}$. Find the matrix product AB .

$$AB = \begin{bmatrix} -1 & 3 \\ 13 & -1 \\ 6 & -2 \end{bmatrix} \quad \checkmark \checkmark$$

- [2 marks] 17. Let $A = \begin{bmatrix} 4 & 5 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 0 \\ 8 & -2 \end{bmatrix}$. Find $A - 3B$.

$$\begin{aligned} A - 3B &= \begin{bmatrix} 4 & 5 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 21 & 0 \\ 24 & -6 \end{bmatrix} \quad \checkmark \\ &= \begin{bmatrix} -17 & 5 \\ -21 & 6 \end{bmatrix} \quad \checkmark \end{aligned}$$

[3 marks] 18. Find the derivative of $y = \sqrt{\log x}$. Simplification is not required.

$$\frac{dy}{dx} = \frac{1}{2} (\log x)^{-1/2} \frac{1}{x \ln 10}$$

chain rule ✓

[3 marks] 19. Find the derivative of $f(p) = \frac{p^2 + 3p}{2p + 1}$. Simplification is not required.

$$f'(p) = \frac{(2p+3)(2p+1) - (p^2+3p)(2)}{(2p+1)^2}$$

quotient rule ✓

[6 marks] 20. Find the equation (in point-slope form) of the tangent line to the curve $f(x) = e^{2x-4}(x^3 - 3)$ at the point (2,5).

$$f'(x) = e^{2x-4} (2)(x^3-3) + e^{2x-4} (3x^2)$$

product rule ✓

$$\begin{aligned} f'(2) &= e^0 (2)(5) + e^0 (12) \\ &= 10 + 12 = 22 \end{aligned}$$

$$y - 5 = 22(x - 2)$$