Fall Term, 2013

Name: Solutions

Section: _____

Tutorial Section: _____

(If you don't remember your tutorial section, write the day and time)

WILFRID LAURIER UNIVERSITY

Waterloo, Ontario

Mathematics 129 – Introductory Calculus

Midterm – October 25, 2013

Instructor:

Section A – 11:30 am MWF – *S. Bauman*
Section B – 12:30 pm MWF – *S. Bauman*

Time Allowed: *80 minutes*

Total Value: *70 marks*

Number of Pages: *6 plus cover page*

Instructions:

Non-programmable, non-graphing calculators are permitted. No other aids are allowed.

Check that your test paper has no missing, blank, or illegible pages.

Answer in the spaces provided. ***Please note that questions are printed on both sides of the test pages.***

Show all your work. Insufficient justification will result in a loss of marks.

Student Number: _____

[4 marks] **1.** Solve the equation $\frac{y}{y-1} = \frac{5}{y} + \frac{9-8y}{y^2-y}$.

multiply by y^2-y: $y(y-1)$

$y^2 = 5(y-1) + 9 - 8y$ ✓

$y^2 = 5y - 5 + 9 - 8y$

$y^2 + 3y - 4 = 0$ ✓

$(y+4)(y-1) = 0$

$y = -4, 1$ ✓

$\therefore y = -4$ inadmissible ✓

[4 marks] **2.** Simplify the expression $3x^3y^2\left(\frac{2x}{y^3}\right)^2$. Write your answer using only positive exponents.

$3x^3y^2\left(\frac{4x^2}{y^6}\right)$ ✓✓

$= \frac{12x^5}{y^4}$ ✓✓

[5 marks] **3.** Solve the inequality $x^4 - x^3 - 2x^2 > 0$. State your answer using interval notation.

$x^4 - x^3 - 2x^2 = 0$

$x^2(x^2 - x - 2) = 0$

$x^2(x-2)(x+1) = 0$ ✓

$x = 0, -1, 2$ ✓

		-1		0		2	
x^2	+	+	+	0	+	+	+
$x-2$	−	−	−	−	−	0	+
$x+1$	−	0	+	+	+	+	+
$x^2(x-2)(x+1)$	+	0	−	0	−	0	+

✓✓

OR

Let $f(x) = x^2(x-2)(x+1)$

$f(-2) = 16$ $f(-\frac{1}{2}) = \frac{-5}{16}$ $f(1) = -2$ $f(2) = 36$ ✓✓

-1		0		2	
+	0	−	0	−	0	+

$\therefore x \in (-\infty, -1) \cup (2, \infty)$ ✓

[*5 marks*] **4.** Solve the equation $2\ln x - \ln 3 = 1$. Give your answer(s) rounded to 2 decimal places.

$2\ln x - \ln 3 = 1$ ✓

$\ln\left(\frac{x^2}{3}\right) = 1$ ✓✓

$e = \frac{x^2}{3}$ ✓

$3e = x^2$ ✓

$x = \sqrt{3e}, -\sqrt{3e}$

$x \approx 2.86$

↑ inadmissible since $x > 0$ ✓

OR

$2\ln x = 1 + \ln 3$

$\ln x = \frac{1+\ln 3}{2}$ ✓✓

$x = e^{(1+\ln 3)/2}$ ✓✓

$x \approx 2.86$ ✓

[*6 marks*] **5.** Solve the system.

$$\begin{aligned} 2y - 6z &= -26 \\ x - 2y + z &= 0 \\ 2x - 5y + 5z &= 18 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 0 & 2 & -6 & -26 \\ 1 & -2 & 1 & 0 \\ 2 & -5 & 5 & 18 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -6 & -26 \\ 2 & -5 & 5 & 18 \end{array}\right] \quad R_1 \leftrightarrow R_2 \quad \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & -6 & -26 \\ 0 & -1 & 3 & 18 \end{array}\right] \quad -2R_1 + R_3$$

✓ ✓ ✓ ✓

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -3 & -13 \\ 0 & -1 & 3 & 18 \end{array}\right] \quad \frac{1}{2}R_2 \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -3 & -13 \\ 0 & 0 & 0 & 5 \end{array}\right] \quad R_2 + R_3$$

✓ ✓

$0 \neq 5$ ✓

$\therefore$ no solution

[*2 marks*] **6.** Complete the following tables of values so that the first table represents a function and the second table does not.

(a)

x	f(x)
1	1
2	2
3	3
4	4
3	3

these must be equal

(b)

x	f(x)
1	1
2	2
3	3
4	4
3	5

these must not be equal

✓ ✓

[4 marks] **7.** Find the interest earned on \$3400 invested for 6 years at 1.5% annual interest compounded continuously.

$A = Pe^{rt}$ ✓

$A = 3400\, e^{0.015(6)}$ ✓

$A = 3720.19$ ✓

Interest $= 3720.19 - 3400 = \$320.19$ ✓

[4 marks] **8.** Find the point-slope form of the equation of the line passing through $(-1, 2)$ which is perpendicular to the line $x + 3y = 4$.

$x + 3y = 4$

$y = \frac{-x}{3} + \frac{4}{3}$

slope $= -\frac{1}{3}$ ✓

∴ slope of desired line is 3 ✓

$y - 2 = 3(x+1)$ ✓✓

[4 marks] **9.** Using interval notation, give the domain of the function defined by $f(x) = \ln(x+3) - \frac{1}{x} + \sqrt{5-x}$

$x + 3 > 0$ $\quad x \neq 0$

$x > -3$ ✓

✓

$5 - x \geq 0$

$x \leq 5$ ✓

domain $= (-3, 0) \cup (0, 5]$ ✓

[2 marks] **10.** Find the average rate of change of $f(w) = 2^{3w}$ over the interval $[1, 4]$. Round your answer to two decimal places.

$AROC = \frac{f(4) - f(1)}{4 - 1}$ ✓

$= \frac{2^{12} - 2^3}{3} \approx 1362.67$ ✓

[*3 marks*] **11.** Find the vertex of the parabola $y = 3x^2 - 24x + 53$.

$y = 3(x^2 - 8x + 16) + 53 - 48$ ✓

$y = 3(x-4)^2 + 5$ ✓

$\therefore$ vertex is $(4, 5)$ ✓

OR vertex $= \left(-\frac{b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ ✓

$= \left(\frac{24}{6}, f(4)\right)$

$= (4, 5)$ ✓ ✓

[*5 marks*] **12.** If $f(x) = 4\sqrt{x}$, using the definition of the derivative, find $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$ ✓

$$= \lim_{h\to 0} \frac{4\sqrt{x+h} - 4\sqrt{x}}{h} \cdot \frac{4\sqrt{x+h} + 4\sqrt{x}}{4\sqrt{x+h} + 4\sqrt{x}}$$ ✓

$$= \lim_{h\to 0} \frac{16(x+h) - 16x}{h(4\sqrt{x+h} + 4\sqrt{x})}$$ ✓

$$= \lim_{h\to 0} \frac{16\cancel{h}}{\cancel{h}(4\sqrt{x+h} + 4\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{16}{4\sqrt{x+h} + 4\sqrt{x}}$$ ✓

$$= \frac{16}{8\sqrt{x}} = \frac{2}{\sqrt{x}}$$ ✓

[*4 marks*] **13.** Solve the equation $3^x = 6^{x+2}$. Give your answer(s) rounded to 2 decimal places.

$3^x = 6^{x+2}$

$\ln(3^x) = \ln(6^{x+2})$ ✓

$x \ln 3 = (x+2)\ln 6$ ✓

$x \ln 3 = x \ln 6 + 2\ln 6$

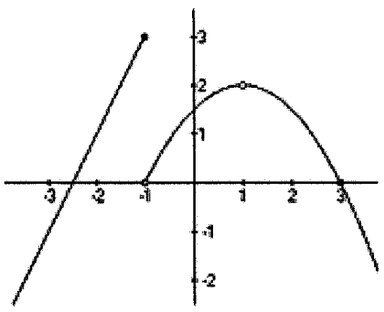
$x \ln 3 - x \ln 6 = 2\ln 6$

$x(\ln 3 - \ln 6) = 2\ln 6$ ✓

$x = \frac{2\ln 6}{\ln 3 - \ln 6}$

$x \approx -5.17$ ✓

[*3 marks*] **14.** Determine the values of (a) $\lim_{x\to -1^+} f(x)$ and (b) $\lim_{x\to -1} f(x)$, (c) $\lim_{x\to 1} f(x)$, if they exist, where $f(x)$ is represented in the figure below.



14.(a) Answer: 0

(b) Answer: DNE

(c) Answer: 2

[*2 marks*] **15.** If $f(x) = x + 3$ and $f(g(x)) = e^x + 4$, what is $g(x)$?

$f(g(x)) = e^x + 4$ $\quad f(g(x)) = g(x) + 3$ ✓

$e^x + 4 = g(x) + 3$

$g(x) = e^x + 1$ ✓

[*2 marks*] **16.** Let $A = \begin{bmatrix} 1 & 3 \\ 6 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 3 \\ -1 & 1 & 5 \end{bmatrix}$. Find the matrix product AB.

$AB = \begin{bmatrix} -1 & 3 & 18 \\ 13 & -1 & 13 \end{bmatrix}$ ✓✓

[*2 marks*] **17.** Let $A = \begin{bmatrix} 4 & 5 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 0 \\ 8 & -2 \end{bmatrix}$. Find $2A - B$.

$2A - B$

$= \begin{bmatrix} 8 & 10 \\ 6 & 0 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 8 & -2 \end{bmatrix}$ ✓

$= \begin{bmatrix} 1 & 10 \\ -2 & 2 \end{bmatrix}$ ✓

[*3 marks*] **18.** Find the derivative of $y = \sqrt[3]{5 \ln x}$. Simplification is not required.

$$\frac{dy}{dx} = \frac{1}{3}(5\ln x)^{-2/3}\left(\frac{5}{x}\right)$$

chain rule ✓

[*3 marks*] **19.** Find the derivative of $f(p) = \dfrac{12^p}{p^2+2p+3}$. Simplification is not required.

$$f'(p) = \frac{12^p \ln 12\,(p^2+2p+3) - 12^p(2p+2)}{(p^2+2p+3)^2}$$

quotient rule ✓

[*3 marks*] **20.** Find the derivative of $f(x) = e^{5x}\log_2 x$. Simplification is not required.

$$f'(x) = e^{5x}(5)\log_2 x + e^{5x}\frac{1}{x\ln 2}$$

product rule ✓