



MAT 1341E Midterm 2, 2011

19-November, 2011.

Instructor: Sanghoon Baek

Family Name: _____

First Name: _____

Student number: _____

Multiple choice answers → {

For the marker's use only → {

1	
2	
3	
4	
5	
6	
[Bonus] 7	
Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this exam. Read each question carefully.
2. This is a closed book exam, and no notes of any kind are allowed. **The use of calculators, cell phones, pagers or any text storage or communication device is not permitted.**
3. Questions 1 to 3 are multiple choice. These questions are worth 1 point each and no part marks will be given. Please record your answers in the space provided above.
4. Questions 4 – 6 require a complete solution, and are worth 6 points each, so spend your time accordingly.
5. Question 7 is a bonus question and is worth 4 points. To earn points here will be more difficult than in questions 1-6.
6. **The correct answer in questions 4–7 requires justification written legibly and logically: you must convince the marker that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
7. Where it is possible to check your work, do so.
8. Good luck! Bonne chance!

1. For what value of α is the set of vectors $\{(1, 1, 1), (1, 2, 0), (2, 3, \alpha)\}$ linearly dependent?

- A. -1
- B. 2
- C. 0
- D. 1
- E. $-1/2$
- F. -2

2. If A is a 7×12 matrix, what is the smallest possible dimension of $\text{null}(A)$?

- A. 0
- B. 3
- C. 5
- D. 7
- E. 11
- F. 12

3. If $B = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 3 & 2 \end{bmatrix}$, then the second row of B^{-1} is:

A. $[1 \ 0 \ -1]$

B. $[-1 \ 0 \ 1]$

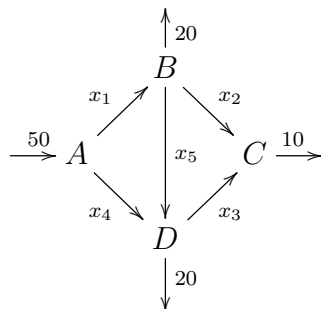
C. $[0 \ 1 \ -1]$

D. $[2 \ 0 \ -1]$

E. $[1 \ -1 \ 0]$

F. None of the above

4. Consider the network of streets with intersections A, B, C and D below. The arrows indicate the direction of traffic flow along the one way streets, and the numbers refer to the number of cars observed to enter A or leave B, C and D during one minute. Each x_i denotes the unknown number of cars which passed along the indicated streets during the same period.



- (a) Write down the linear system which describes the traffic flow, **together with all the constraints** on the variables x_i , $i = 1, \dots, 5$. (Do not perform any operations on your equations: this is done for you in (b). *Do not simply copy out the equations implicit in (b). You will not get any marks if you do this.*)

(b) The reduced row-echelon form of the augmented matrix from part (a) is

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 0 & 50 \\ 0 & 1 & 0 & 1 & 1 & 30 \\ 0 & 0 & 1 & -1 & -1 & -20 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Give the general solution. (Ignore the constraints from (a) at this point.)

(c) If \overline{BD} is closed, find the maximum and minimum flows along \overline{BC}

5. Let $W = \text{span}\{(1, 0, 1, 1), (-1, 1, 2, 0), (1, 1, 4, 2), (0, 1, 3, 1)\}$, and define a matrix A by

$$A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 2 & 4 & 3 \\ 1 & 0 & 2 & 1 \end{bmatrix}$$

- (a) Find a basis for W which is a subset of the given spanning set.
- (b) Extend your basis in part (a) to a basis of \mathbf{R}^4 .
- (c) Find a basis for $\text{null}(A)$ and hence find its dimension.
- (d) Extend your basis of $\text{null}(A)$ to a basis of \mathbf{R}^4 .

Space for Question 5

6. Suppose A is an $n \times n$ matrix and that there is a vector $b \in \mathbf{R}^n$, for which $Ax = b$ is inconsistent.

State whether each of the following is (always) true, or is (possibly) false, in the box after the statement.

- If you say the statement may be false, you must give an explicit example (with numbers!).
- If you say the statement is true, you must give a clear explanation.

(a) The system $Ax = 0$ has a unique solution.

(b) The matrix A is not invertible.

(c) The rows of A are linearly independent.

7. **[Bonus]**. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, and suppose that $ad - bc = 0$. Show carefully that A is not invertible.