CONCORDIA UNIVERSITY
Faculty of Engineering and Computer Science
Department of Mechanical and Industrial Engineering

MECH 370 - Modelling, Simulation and Control Systems

SOLUTIONS - ASSIGNMENT # 1

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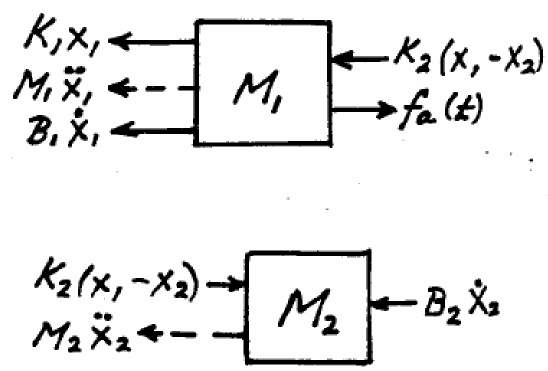
2.1 Summing the forces shown on each of the free-body diagrams, we have

$$M_1\ddot{x}_1 + B_1\dot{x}_1 + K_1x_1 + K_2(x_1 - x_2) = f_a(t)$$
$$M_2\ddot{x}_2 + B_2\dot{x}_2 - K_2(x_1 - x_2) = 0$$

Collecting terms gives

$$M_1\ddot{x}_1 + B_1\dot{x}_1 + (K_1 + K_2)x_1 - K_2x_2 = f_a(t)$$
$$-K_2x_1 + M_2\ddot{x}_2 + B_2\dot{x}_2 + K_2x_2 = 0$$

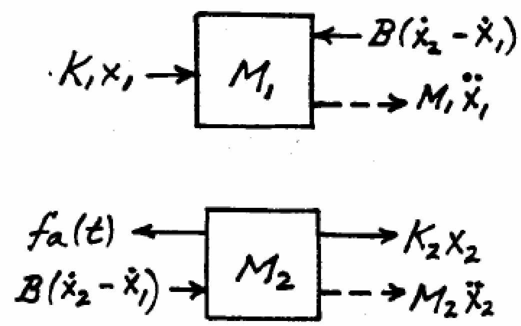


2.2 Summing the forces shown on each of the free-body diagrams, we have

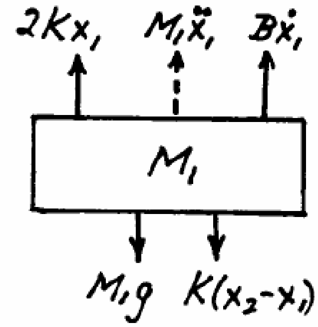
$$M_1\ddot{x}_1 + B(\dot{x}_1 - \dot{x}_2) + K_1x_1 = 0$$
$$M_2\ddot{x}_2 + B(\dot{x}_2 - \dot{x}_1) + K_2x_2 = f_a(t)$$

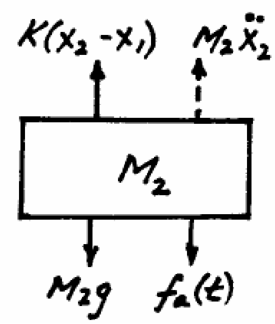
Collecting terms gives

$$M_1\ddot{x}_1 + B\dot{x}_1 + K_1x_1 - B\dot{x}_2 = 0$$
$$-B\dot{x}_1 + M_2\ddot{x}_2 + B\dot{x}_2 + K_2x_2 = f_a(t)$$



2.16 (a) From the free-body diagrams,





$$\begin{aligned} M_1\ddot{x}_1 + B\dot{x}_1 + 3Kx_1 - Kx_2 &= M_1 g \\ -Kx_1 + M_2\ddot{x}_2 + Kx_2 &= M_2 g + f_a(t) \end{aligned}$$

(b) With x_1 and x_2 replaced by the constant displacements x_{1_0} and x_{2_0}, and with $f_a(t) = 0$,

$$3x_{1_0} - x_{2_0} = \frac{M_1 g}{K} \qquad \text{and} \qquad -x_{1_0} + x_{2_0} = \frac{M_2 g}{K}$$

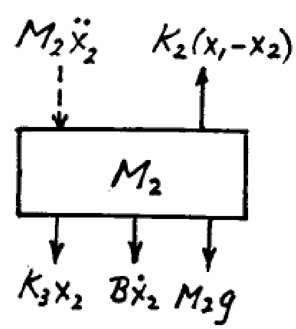
from which

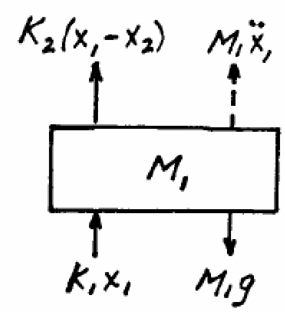
$$x_{1_0} = (M_1 + M_2)\frac{g}{2K} \qquad \text{and} \qquad x_{2_0} = (M_1 + 3M_2)\frac{g}{2K}$$

(c) Substituting $x_1 = x_{1_0} + z_1$ and $x_2 = x_{2_0} + z_2$ into the original differential equations, using the above expressions for x_{1_0} and x_{2_0}, and canceling common terms, we obtain

$$\begin{aligned} M_1\ddot{z}_1 + B\dot{z}_1 + 3Kz_1 - Kz_2 &= 0 \\ -Kz_1 + M_2\ddot{z}_2 + Kz_2 &= f_a(t) \end{aligned}$$

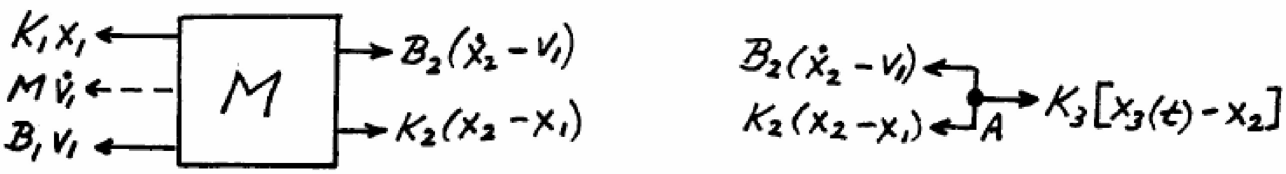
2.22 When drawing the free-body diagrams, note that the upward force of the cable on M_2 is the same as the upward force of the cable on M_1 because of the pulley. Summing the forces shown on each of the diagrams and collecting terms, we get





$$\begin{aligned} M_1\ddot{x}_1 + (K_1 + K_2)x_1 - K_2x_2 &= M_1 g \\ -K_1x_1 + M_2\ddot{x}_2 + B\dot{x}_2 + (K_2 + K_3)x_2 &= -M_2 g \end{aligned}$$

3.13



We sum the forces shown on each of the free-body diagrams, where $v_1 = \dot{x}_1$.

$$\begin{aligned} M\dot{v}_1 + (B_1 + B_2)v_1 + (K_1 + K_2)x_1 - B_2\dot{x}_2 - K_2x_2 &= 0 \\ -B_2v_1 - K_2x_1 + B_2\dot{x}_2 + (K_2 + K_3)x_2 &= K_3x_3(t) \end{aligned}$$

Solving these equations for $\dot{v}_1$ and $\dot{x}_2$, we have

$$\dot{x}_1 = v_1 \quad \text{(A)}$$

$$\dot{v}_1 = \frac{1}{M}[-(K_1 + K_2)x_1 - (B_1 + B_2)v_1 + K_2x_2 + B_2\dot{x}_2] \quad \text{(B)}$$

$$\dot{x}_2 = \frac{1}{B_2}[K_2x_1 + B_2v_1 - (K_2 + K_3)x_2 + K_3x_3(t)] \quad \text{(C)}$$

Substituting (C) into (B), in order to eliminate the derivative on the right side, and simplifying the result, we obtain

$$\dot{v}_1 = \frac{1}{M}[-K_1x_1 - B_1v_1 - K_3x_2 + K_3x_3(t)] \quad \text{(D)}$$

The state-variable equations are (A), (C), and (D). The output is given by $y = B_2(\dot{x}_2 - v_1)$. Inserting (C) into this expression and simplifying the result give the output equation

$$y = K_2x_1 - (K_2 + K_3)x_2 + K_3x_3(t)$$

3.28 We define

$$\mathbf{q} = \begin{bmatrix} x_1 \\ q \end{bmatrix}, \qquad \mathbf{u} = x_2(t), \qquad \text{and} \qquad \mathbf{y} = \begin{bmatrix} x_1 \\ v_1 \end{bmatrix}$$

Note that $n = 2$, $m = 1$, and $p = 2$. In this problem, care must be taken to distinguish q, which is the second state variable, from the state vector $\mathbf{q}$. From (21) and (22), we have

$$\dot{\mathbf{q}} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\left(\frac{K_1 + K_2}{M_1}\right) & -\frac{B}{M_1} \end{bmatrix}}_{\mathbf{A}} \mathbf{q} + \underbrace{\begin{bmatrix} B/M_1 \\ \frac{K_2}{M_1} - \frac{B^2}{M_1^2} \end{bmatrix}}_{\mathbf{B}} \mathbf{u}$$

$$\mathbf{y} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{C}} \mathbf{q} + \underbrace{\begin{bmatrix} 0 \\ B/M_1 \end{bmatrix}}_{\mathbf{D}} \mathbf{u}$$