Disjunctive Normal Form

- An **atomic** proposition is a proposition containing no logical connectives.

- A **literal** is either an atomic proposition or a negation of an atomic proposition.

- A **conjunctive clause** is a proposition that contains only literals and (possibly) the connective $\wedge$ such that no atomic proposition appears more than once.

- A proposition in said to be in **disjunctive normal form (DNF)** if it is a disjunction of conjunctive clauses.

- **How to find a DNF using truth tables:** Suppose you wish to find a DNF for a compound proposition P that involves atomic propositions $p_1, \ldots, p_n$.

  E.g. $P \equiv (p_1 \to p_2) \leftrightarrow (p_1 \wedge p_2)$.

  1. Write a truth table for P. Note that it will have 2^n rows corresponding to the 2^n possible truth assignments for $p_1, \ldots, p_n$.

     E.g.

p_1	p_2	$(p_1 \to p_2) \leftrightarrow (p_1 \wedge p_2)$
T	T	T
T	F	T
F	T	F
F	F	F

  2. Identify the rows that result in truth value T for P. Write a conjunctive clause corresponding to each of these rows.

     E.g.

p_1	p_2	$(p_1 \to p_2) \leftrightarrow (p_1 \wedge p_2)$	Conjunctive clause
T	T	T	$p_1 \wedge p_2$
T	F	T	$p_1 \wedge \neg p_2$

  3. The disjunction of these conjunctive clauses is a DNF for P.

     E.g. $(p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2)$

- **Facts:**
  - Every proposition is equivalent to a proposition in disjunctive normal form.
  - DNF is not unique. All DNFs for the same proposition are mutually equivalent.

    E.g. $(p_1 \wedge p_2) \vee (p_1 \wedge \neg p_2)$ and p_1 are both DNFs for $(p_1 \to p_2) \leftrightarrow (p_1 \wedge p_2)$.
  - The truth table method (without any simplification) leads to a unique DNF, except for the order of conjunctive clauses, and the order of literals in each conjunctive clause.
  - A DNF can also be obtained using other methods, e.g. using truth trees or the table of logical equivalences.