Assignment #1
Linear Programming
Formulation, Graphical Method and Excel Solver

Solution and Marking Scheme

20 points

After inputting the final grade on the 1st page, if the statement of integrity is not available or not signed by the given student (e.g. a team of three, two signed but the third did not) then deduct 2 marks. Show the "original grade" – 2 = "new grade".

1. Please don't forget to show the grade subtracted next to each mistake and show the correct answer.
2. Show that the grade is out of 10 (for example 24/30). Also please do write how much they got on each problem (i.e. **5/6.5** for problem 1 etc…)
3. Please let me know the most common mistakes so I can go over them in class.

Problem 1 (6.5 points):

Solution

1. **Algebraic formulation (2 points)**

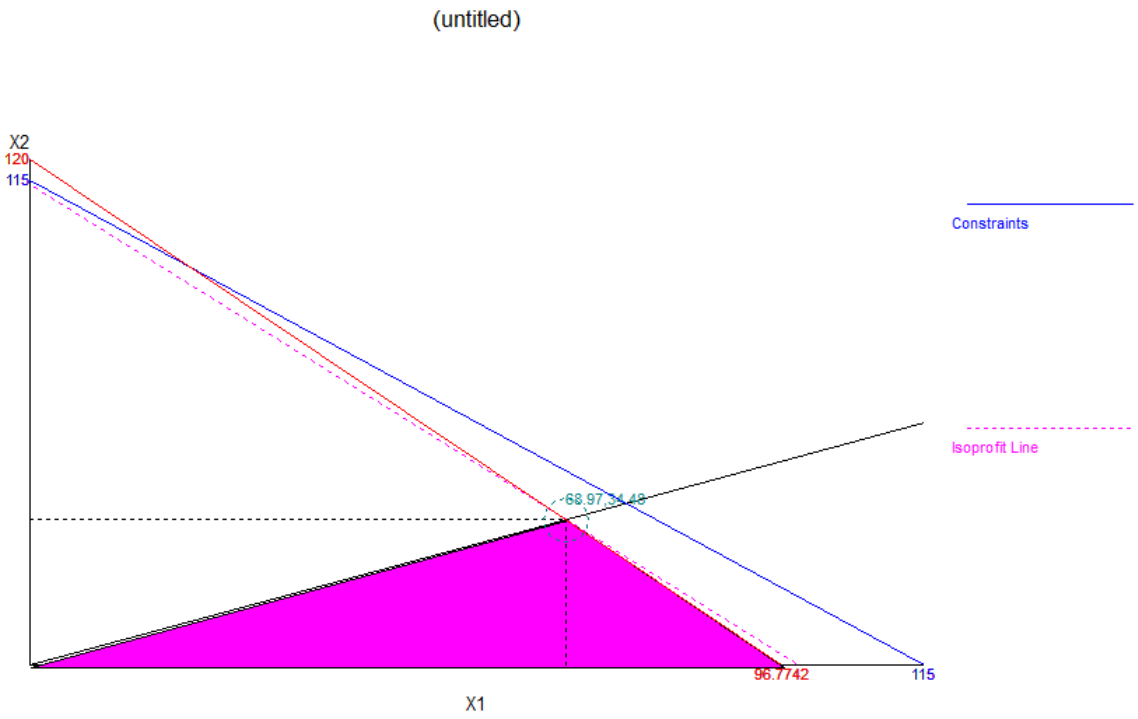
0.5 point: for decision variables and their definition (**deduct the whole mark** if the definition is incomplete for example *X1*= ice cream and X2= frozen yougurt)
0.25 point: objective function
1.25 point: Constraints

1. Let X1 = number of gallons of ice cream to order each week.
   X2 = number of gallons of frozen yogurt to order each week.

   Maximise Z = $4.15X1 + 3.60 X2 (Profit, $)
   Subject to
   (1) X1 + X2 <= 115 (freezer space constraint, gallons) (0.25 point)
   (2) $0.93X1 + $0.75X2 <= 90 (budget constraint, $) (0.25 point)
   (3) X1 – 2X2 >= 0 (demand constraints) (0.75 point)

2. **Graphical method solution (3 points)**

1.5 point: treat the constraint correctly. (0.5 point for each constraint)
0.5 point: find the feasible region
0.5 point: The student has to demonstrate that he/she used either the isoprofit method (drawing the objective function line) OR the corner point method (by checking all the three corner points and picking the one that leads to the largest value for Z)
0.5 point: Solve algebraically constraint equation (2) and constraint equation (3) for the optimal solution: X1 = 68.97, X2 = 34.48 and Z = $410.35



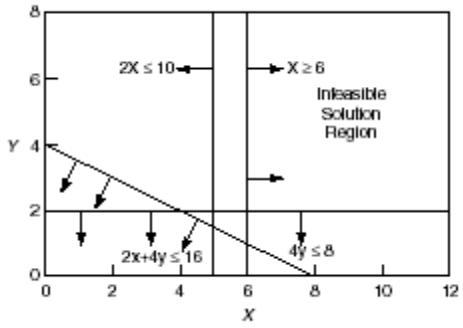
3. **(0.5 point)** **Managerial statement:** In order to maximize profit, Gillian's Restaurant should order approximately 68.96 or 68.97 gallons of ice cream and 34.48 gallons of frozen yogurt for a maximum profit of $410.35.
   P.S.: if the student uses 68.96 gallons of ice cream then Z= $410.31

4. **(1 point)**: **NO PARTIAL marks if the student does not provide a justification or if the student provides a wrong reasoning that lead to correct conclusion.**
   Answer: No additional profit, as the freezer space constraint (1) is NOT a "binding constraint" (i.e. constraint (1) is "non binding", which mean you have a slack and if you increase the capacity to 135 you will have more slack.)
   No additional profit, as the change in freezer space constraint (1) has no impact of the feasible region, and thus the optimal solution remain the same.
   P.S.: The freezer space, constraint (1), is also a redundant constraint. A redundant constraint is also a non binding constraint.

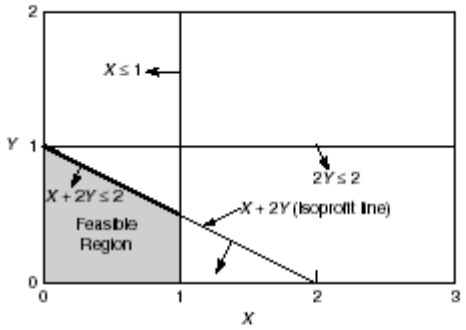
Problem 2: (4 points)

1 point for each Formulation (Deduct 0.25 point per mistake for a maximum of four mistakes)
The student for each formulation must show his/her work (i.e. plot the graph and show how he/she concluded the LP special case) otherwise the student will get ZERO point.

Formulation 1:



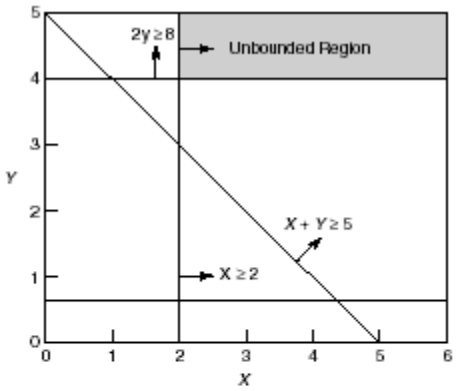
Formulation 2:



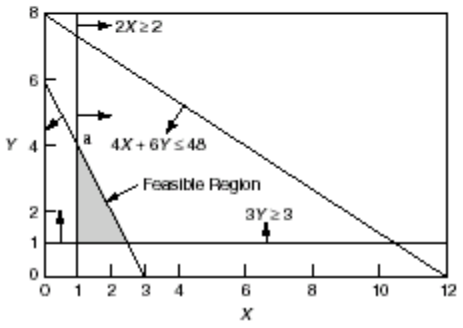
$X + 2Y = 2$ line—this is also on the same slope as the isoprofit line $X + 2Y$ and hence there will be more than one optimal solution.

As a matter of fact, every point along the heavy line will provide an "alternate optimum."

Formulation 3:



Formulation 4:



Formulation 4 has a unique optimal solution (X = 1 and Y = 1). Note that the constraint 4X+6Y<= 48 is redundant.

Problem 3(4.5 points)

1. Formulate algebraically a linear programming model for this problem in order to determine how many medical beds and how many surgical beds should be added to maximize revenues. Define the decision variables, objective function, and constraints. **(2.5 points)**

Marking Scheme

- **0.5 point** decision variables definition.
- **0.5 point** objective function
- **1.5 points**: **the first constraint is 0.5 point (i.e. patient-days available)**
  All the other constraints (including the non-negativity) are **0.25 point**

3-14. M = number of medical patients

S = number of surgical patients

Maximize revenue = $\$2,280M + \$1,515S$

subject to

$$8M + 5S \leq 32{,}850 \text{ (patient-days available} = 365 \text{ days} \times 90 \text{ new beds)}$$

$$3.1M + 2.6S \leq 15{,}000 \text{ (lab tests)}$$

$$1M + 2S \leq 7{,}000 \text{ (x-rays)}$$

$$S \leq 2{,}800 \text{ (operations/surgeries)}$$

$$M, S \geq 0$$

Solution: (See file P3-14.XLS)

M = 2,791 medical patients

S = 2,105 surgical patients

revenue = $9,551,659 per year

To convert M and S to number of medical versus surgical beds, find the total number of hospital days for each type of patient:

medical = (2,791 patients)(8 days/patient)

= 22,328 days

2. Formulate this same linear programming problem on a spreadsheet and SOLVE using Excel solver (Provide a printout of the corresponding "Excel Spreadsheet" and the "Answer Report"). **(1.5 points)**

   Please check the Excel spreadsheet for formulation and Solution.

3. Describe clearly and completely the optimal solution to this problem using a managerial statement. **(0.5 point)**

Managerial statement: the best patients' mix maximizing revenues from the 90-beds addition is 2790.909 medical patients (approximately 2,791 medical patients) 2104.545 surgical patients (approximately 2,104 surgical patients) for optimal revenue of $9,551,659 per year.

P.S.: Accept also revenue that is based on the rounding solution.

Problem 4 (5 points)

The formulation for this problem could take several forms. Please check the Excel spreadsheet for formulation, solution and managerial statement.

1. Formulate the algebraic LP formulation whose objective is to maximize the net revenue from the sale of the sauces. **(3 points)**

    - **0.75 point** decision variables definition,
    - **0.75 point** objective function
    - **1.5 points** for the constraints (including the non-negativity). Subtract 0.25 per wrong constraints

2. Find the optimal solution using Solver. (Provide a printout of the Answer report and the Excel spreadsheet formulation).

    Excel Spreadsheet + Answer Report **(1.5 points)**

3. Recommendation/managerial statement **(0.5 point)**

Source:

Problem 1: Taylor, B.W. III 2007. *Introduction to Management Science*. (9th ed.) Prentice-Hall, Upper Saddle River, New Jersey. 771p. + appendices.

Problem 2 and Problem 3: Render, B., and R.M. Stair, Jr., N. Balakrishnan 2003. *Managerial Decision Modeling*. Prentice-Hall, Inc.: Upper Saddle River, New Jersey. 616p.