

MAT 1322 MIDTERM 1, VERSION A

1. [2 points, 5.10 #19] Determine if the integral $\int_2^\infty \frac{1}{x(\ln x)^2} dx$ is convergent or divergent and evaluate if it is convergent.

A.1 B.2.51 C.0.50 D.5.33 E.1.44 F.divergent

Solution.

$$\int_2^\infty \frac{1}{x(\ln x)^2} dx = \int_{\ln 2}^\infty \frac{1}{u^2} du \quad [u = \ln x]$$
$$= \left[-\frac{1}{u}\right]_{\ln 2}^\infty = 0 + \frac{1}{\ln 2} \doteq 1.44$$

2. [2 points, 6.1 #11] Find the area of the region enclosed by the curves $y^2 = 2x$ and $2x - 2y = 3$.

A.5.33 B.4.77 C.3.55 D.6 E.9.01 F.2.22

Solution. Intersection $y^2 = 2x$ and $2x - 2y = 3 \Rightarrow y^2 - 2y - 3 = 0 \Rightarrow (y - 3)(y + 1) = 0 \Rightarrow y = -1, 3$.

$$A = \int_{-1}^3 \left[\left(\frac{1}{2}(2y + 3)\right) - \frac{1}{2}y^2 \right] dy = \frac{1}{2} \int_{-1}^3 (2y + 3 - y^2) dy = \frac{1}{2} \left[y^2 + 3y - \frac{1}{3}y^3 \right]_{-1}^3 = \frac{1}{2} \frac{32}{3} = 5.33$$

3. [2 points, 6.2 #13] The region enclosed by the curves $y = \sqrt[3]{x}$, $x = 1$, $y = 0$ is rotated about the line $x = 2$. Find the volume of the resulting solid.

A.11.44 B.7.17 C.5.10 D.12.57 E.10.11 F.6.73

Solution. $y = \sqrt[3]{x} \Rightarrow x = y^3$

$$A = \int_0^1 \pi[(y^3 - 2)^2 - 1^2] dy = \frac{15}{7}\pi \doteq 6.73$$

4. [2 points, 6.2 #31] The base of a solid is the region $\{(x, y) \mid x^2 \leq y \leq 2\}$. Cross-sections perpendicular to the y -axis are semicircles. Find the volume of the solid.

A.6 B.3.14 C.8.02 D.11.50 E.7.07 F.4.75

Solution. The semicircle with base though (x, y) on the curve $y = x^2$ has radius $r = x$. Thus $r^2 = x^2 = y$ and

$$A = \int_0^2 \frac{1}{2}\pi r^2 dy = \int_0^2 \frac{1}{2}\pi y dy = \left[\frac{1}{4}\pi y^2\right]_0^2 = \pi = 3.14$$

5. [2 points, 6.3 #7] Find the arclength of the curve $y = 1 + x^{3/2}$ from $x = 0$ to $x = 4$.

A.12.50 B.9.07 C.24.24 D.10.61 E.20 F.14.25

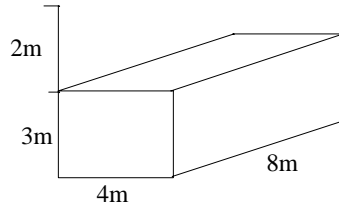
Solution. $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

$$\frac{dy}{dx} = \frac{3}{2}x^{1/2}, \left(\frac{dy}{dx}\right)^2 = \frac{9}{4}x$$

$$L = \int_0^4 \sqrt{1 + \frac{9}{4}x} dx \quad [u = 1 + \frac{9}{4}x, du = \frac{9}{4}dx]$$

$$= \int_1^{10} u^{1/2} \frac{4}{9} du = \frac{4}{9} \left[\frac{2}{3}u^{3/2}\right]_1^{10} \doteq 9.07$$

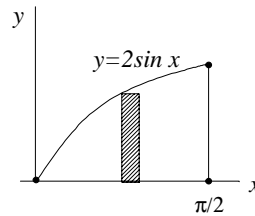
6. [2 points, 6.5 #13] The rectangular tank shown is full of water. Find the work required to pump the water up 2m above its top. (The density of water is 1000kg/m^3 and the acceleration of gravity 9.8m/sec^2 . Work in joules J.)
 A.3263, 500 B.4939, 200 C.4992, 240 D.3292, 800 E.5103, 450 F.6485, 200



Solution. The work required to pump a slice at a distance x from the 2m above the tank is $(1000 \times 9.8)(4 \times 8)x\Delta x = 313,600 \text{ J}$. The total work is $W = \int_2^5 (1000 \times 9.8)(4 \times 8)xdx = 3292,800 \text{ J}$

7. [3 points, 6.5 #29] Consider the region bounded by the curves $y = 2 \sin x$, $y = 0$, $x = 0$, $x = \pi/2$.
 (a) Sketch the region.
 (b) Find the moment M_y about the y axis if the mass density is ρ (constant).
 (Provide a detailed solution and draw a box around the final answer.)

Solution.



$$\begin{aligned} \Delta M &= \rho y \Delta x, \\ M_y &= \int_0^{\pi/2} x \rho y dx = \rho \int_0^{\pi/2} x 2 \sin x dx \\ &= 2 \int_0^{\pi/2} x \sin x dx \quad [u = x, v' = \sin x, u' = 1, v = -\cos x] \\ &= \rho 2 [-x \cos x + \int \cos x]_0^{\pi/2} = \rho 2 [-x \cos x + \sin x]_0^{\pi/2} = 2\rho \end{aligned}$$