MATH2007B - Test 1 - 14:35 -15:25, Sept 30, Tuesday

Name: Student Number:

Total: 15 marks
Closed book, Non-programmable calculators are allowed!

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1. (4 points) Multiple choices, no partial marks.

   (i) $\lim\limits_{x \to 1} \frac{x^4+2x-3}{x^3-1} =$

   (a) 2 (b) 3 (c) 1/4 (d) 1/2 (e) does not exist.

   Solution: (a). By L'Hospital's Rule, $\lim\limits_{x \to 1} \frac{x^4+2x-3}{x^3-1} = \lim\limits_{x \to 1} \frac{4x^3+2}{3x^2} = \frac{6}{3} = 2.$

   (ii) $\lim\limits_{x \to 0} \frac{\sin(6x)}{\sin(2x)} =$

   (a) 0 (b) 1 (c) 2 (d) 3 (e) does not exist.

   Solution: (d). By L'Hospital's Rule, $\lim\limits_{x \to 0} \frac{\sin(6x)}{\sin(2x)} = \lim\limits_{x \to 0} \frac{6\cos(6x)}{2\cos(2x)} = \frac{6}{2} = 3.$

   (iii) $\lim\limits_{x \to +\infty} x^2 e^{-x} =$

   (a) 0 (b) 1 (c) 2 (d) 4 (e) does not exist.

   Solution: (a). By L'Hospital's Rule, $\lim\limits_{x \to +\infty} x^2 e^{-x} = \lim\limits_{x \to +\infty} \frac{x^2}{e^x} = \lim\limits_{x \to +\infty} \frac{2x}{e^x} = \lim\limits_{x \to +\infty} \frac{2}{e^x} = 0.$

   (iv) $\int_{-1}^{1} (x^3+2x)e^{(x^8+1)}\, dx =$

   (a) 0 (b) 1/8 (c) 1/9 (d) 3 (e) 4

   Solution: (a). Note that the function $(x^3+2x)e^{(x^8+1)}$ is odd and the interval is symmetric about the origin, so the integral is 0.

2. (2 points) Evaluate $\displaystyle\int (6x+3)\sqrt{x^2+x+1}\, dx.$

   Solution: Let $u = x^2+x+1$. Then $du = (2x+1)dx$. Thus

$$\int (6x+3)\sqrt{x^2+x+1}\, dx = \int 3\sqrt{u}\, du = 2u^{3/2} + C = 2(x^2+x+1)^{3/2} + C.$$

3. (3 points) Evaluate $\int 9x^2 \ln x \, dx$.

Solution: By Integration by parts,

$$\int 9x^2 \ln x \, dx = \int 3 \ln x \, dx^3 = 3x^3 \ln x - \int 3x^3 \, d(\ln x)$$
$$= 3x^3 \ln x - \int 3x^3 \frac{dx}{x} = 3x^3 \ln x - \int 3x^2 \, dx$$
$$= 3x^3 \ln x - x^3 + C.$$

4. (2 points) Evaluate $\int 2\cos^2 x \, dx$.

Solution: By the double angle formula, $2\cos^2 x = 1 + \cos(2x)$.

$$\int 2\cos^2 x \, dx = \int [1 + \cos(2x)] dx = x + \frac{1}{2}\sin(2x) + C$$

5. (2 points) Evaluate $\int 2\sin(5x)\cos(3x) \, dx$.

Solution:

$$\int 2\sin(5x)\cos(3x) \, dx = \int [\sin(5x + 3x) + \sin(5x - 3x)] dx = \int [\sin(8x) + \sin(2x)] dx$$
$$= -\frac{1}{8}\cos(8x) - \frac{1}{2}\cos(2x) + C.$$

6. (2 points) Evaluate $\int \sin^3 x \cos^2 x \, dx$.

Solution: We use the trig substitution $u = \cos(x)$, $du = -\sin(x)\,dx$. Then

$$\begin{aligned}
\int \sin^3(x)\cos^2 x\,dx &= \int \sin^2(x)(\cos^2(x))\sin(x)\,dx = \int (1-\cos^2(x))(\cos^2(x))\sin(x)\,dx \\
&= \int (1-u^2)u^2\,(-du) = \int (-u^2+u^4)\,du \\
&= -\frac{1}{3}u^3 + \frac{1}{5}u^5 + c \\
&= -\frac{1}{3}\cos^3(x) + \frac{1}{5}\cos^5(x) + c
\end{aligned}$$