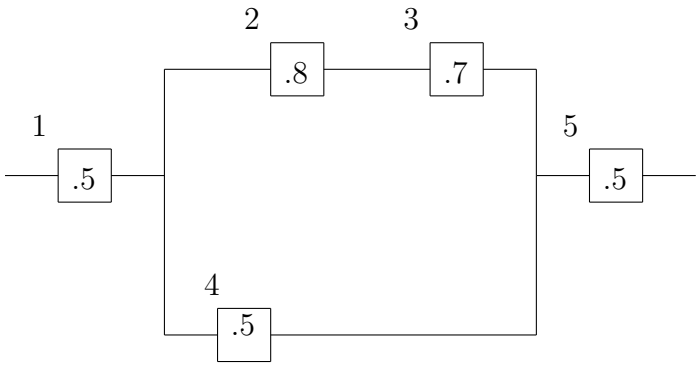
MAT 2377 - Practice Midterm

1. Consider the following circuit with six components. We say that it is functional if there is a path component of functional components from left to right. The probability that the component is functional is indicated. Suppose that the components work or fail independently. What is the probability that the circuit is functional?



A) 0.84 B) 0.16 C) 0.035 D) 0.50 E) 0.195

2. Consider 1000 couples such that both the husband and the wife work. Each person is asked if their salary exceeds $\$30,000$. Here are the results:

	husband (at most $\$30,000$)	husband (more than $\$30,000$)
wife (at most $\$30,000$)	425	400
wife (more than $\$30,000$)	65	110

Determine the probability that the husband has a salary greater than $\$30,000$, given that his wife has a salary greater than $\$30,000$.

A) 0.6286 B) 0.3714 C) 0.4300 D) 0.0650 E) aucune de ces réponses

3. The probability that a student will pass an exam is 0.8. If we consider a group of 20 students, what is the probability that the majority (that is more than half) of the students are going to pass the exam?

   A) 0.0026 B) 0.0100 C) 0.0480 D) 0.9520 E) 0.9974

4. Let X be the number of failures for a certain machine in a month.

$$F_X(x) = \begin{cases} 0, & x < 0 \\ 0.17, & 0 \le x < 1 \\ 0.40, & 1 \le x < 2 \\ 0.59, & 2 \le x < 3 \\ 0.72, & 3 \le x < 4 \\ 0.80, & 4 \le x < 5 \\ 1, & x \ge 5 \end{cases}$$

   Compute the probability that there are more than 2 failures in a month.

   A) 0.60 B) 0.72 C) 0.20 D) 0.80 E) 0.41

5. Refer to Question 4. What is the expected number of failures in a month?

   A) 2.50 B) 3.00 C) 2.32 D) 11.94 E) none of the preceding

6. Suppose that major cracks on the road can be modeled as a Poisson process with 0.2 cracks per kilometer on average. What is the probability that there is at most one major crack found during an inspection of 10 kilometres? Note: $\exp(x) = e^x$.

   A) $3\exp(-2)$ B) $\exp(-2)$ C) $\exp(-2) - \exp(-1)$ D) $1.2\,e^{-0.2}$ E) none of the preceding

7. Two companies A and B consider making an offer on a road construction work. Company A submits a bid and the probability is $4/5$ that the company will get the job, if B does not submit a bid. The probability that B will bid is $1/3$, and if so, the probability that A gets the

job is only 1/3. What is the probability that A will get the job?

A) 0.6444 B) 0.3556 C) 0.7556 D) 0.3333 E) 0.2444

8. Let X be a continuous random variable with the following probability density function:

$$f(x) = \begin{cases} 0, & \text{for } x \leq -1 \\ \frac{3}{4}(1-x^2), & \text{for } -1 < x < 1 \\ 0, & \text{for } x \geq 1 \end{cases}$$

The value of $P(0 < X < 0.75)$ is

A) 0.9141 B) 0.4570 C) 0.3437 D) 0.3960 E) none of the preceding

9. In case of emergency, I need a fully charged battery for my flashlight. I find many batteries that I bought one year ago. Suppose that there is a probability of 0.1 that a battery which is a year old is still fully charged. What is the probability that the third battery I put in my flashlight is the first one that works?

A) 0.081 B) 0.729 C) 0.0432 D) 0.0729 E) none of the preceding

10. A and B are two events such that $P(A) = .3$, $P(B) = .5$ and $P(A \cup B) = .65$. Are A and B independent events?

A) Yes B) No C) possibly, but insufficient information is given

11. Experience has shown that 2% of the welds in a particular type of pipe are defective. A device for checking the welds is designed to send a signal if a defective weld is detected. The device correctly sends a signal 90% of the time when the weld is defective and the device incorrectly sends a signal 5% of the time when if fact the weld is good. If a signal is sent the next time we check a weld, what is the probability that the weld really is defective?

A) 0.4015 B) 0.2687 C) 0.3725 D) 0.4275 E) 0.1283

12. Experience has shown that the grades in MAT 2377 are normally distributed with mean 66 and variance 64. What proportion of the class will get a grade of at least 80?

    (a) 0.9599 (b) 0.5871 (c) 0.4129 (d) 0.0401 (e) none of the preceding