EXAM F

MAT2377 C: Probability and Statistics for Engineers

Instructor: Aziz Khanchi

Midterm Test-Solution

October 2013

Surname ______________ First Name ______________

Student # ______________

Take your time to read the entire paper before you begin to write, and read each question carefully. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- There are two short answer questions and 9 multiple choice questions. Detail of your work is required for short answer questions.
- Do **not** detach the exam booklet. You have to return the complete stapled booklet.
- You have 80 minutes to complete this exam.
- Only answers recorded in the table on the second page will be marked.
- This is an open book exam, and notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted.**
- Where it is possible to check your work, do so.

Good Luck!

Answers for the multiple choice questions should be written in this table.

Question	Answer
1	
2	
3	
4	
5	
6	
7	
8	
9	

[4 pt.] 1. The probability distribution of X, the number of typographical errors per page in online manuscripts are given by

x	0	1	2	3
$f(x)$	3/7	2/7	1/7	c

(i) Find c such that f is a probability mass function.

Solution: Since

$$\sum_x f(x) = 1$$

we get $c = 1/7$.

(ii) Compute (μ), the expected number of typographical errors per page and σ, the standard deviation of number of typographical errors per page.

Solution:

$$E(X) = \sum xf(x) = 0(3/7) + 1(2/7) + 2(1/7) + 3(1/7) = 1.$$

and

$$Var(X) = (0-1)^2(3/7) + (1-1)^2(2/7) + (2-1)^2(1/7) + (3-1)^2(1/7) = 8/7.$$

$$\sigma_X = \sqrt{8/7}$$

(iii) Compute $P(0.3 \le X \le 2.8)$.

Solution: $P(0.3 \le X \le 2.8) = P(X = 1) + P(X = 2) = 3/7$.

(iv) Compute the cumulative distribution function (c.d.f.) for the random variable X.

Solution:

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ 3/7 & \text{if } 0 \le x < 1 \\ 5/7 & \text{if } 1 \le x < 2 \\ 6/7 & \text{if } 2 \le x < 3 \\ 1 & \text{if } 3 \le x \end{cases}$$

[3 pt.] 2. Consider an electronic board with three components 1, 2 and 3. Assume components work independently. Define

$$P(A_i) = P(i^{th} \text{ component is functional}) \quad \text{for} \quad i = 1, 2, 3.$$

Let $P(A_1) = 0.9, P(A_2) = 0.8$ and $P(A_3) = 0.95$.

(i) What is the probability that at least one component is functional?

Solution: $1 - P(A_1' \cap A_2' \cap A_3') = 1 - (1 - 0.9)(1 - 0.8)(1 - 0.95) = 0.999$.

(ii) What is the probability that exactly two components are functional.

Solution:

$$P(A_1' \cap A_2 \cap A_3) + P(A_1 \cap A_2' \cap A_3) + P(A_1 \cap A_2 \cap A_3')$$
$$= 0.1(0.8)(0.95) + 0.9(0.2)(0.95) + 0.9(0.8)(0.05) = 0.283$$

(iii) Find $P(A_1 \cap A_2' \cap A_3)$.

Solution: $P(A_1 \cap A_2' \cap A_3) = 0.9(0.2)(0.95) = 0.171$.

Multiple Choice Questions, 2 points each.

Submit your answers for the multiple choice questions in the table found on the second page.

Question 1. From an announcement by Air Canada it is known that 10% of clients reserve first class seats. Among the next 20 reservations, what is the probability that at least one of them reserves a first class seat if reservations are made independently ?

(a) 0.2701

(b) 0.1216

(c) 0.8784

(d) 0.9865

(e) 0.9999.

Solution: X: number of clients reserving first class. $X \sim Binomial(n = 20, 0.1)$.

$$P(X \geq 1) = 1 - P(X = 0) = 1 - (0.9)^2 0 = 0.8784$$

Question 2. In a communication system there is one error every 10 seconds, on average. If the number of errors have a Poisson distribution, calculate the probability that in 30 seconds we have at least one error.

(a) $1 - 4e^{-3}$

(b) $1 - 2e^{-1}$

(c) $1 - e^{-1}$

(d) $1 - 3e^{-3}$

(e) $1 - e^{-3}$.

Solution: Take unit of time to be 10s.

$$X_1 \sim Poisson(1).$$

Let

$$Y = X_3 = \text{number of errors in 30 seconds.}$$

Then

$$Y \sim Poisson(3).$$

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - \exp(-3)3^0/0! = 1 - \exp(-3).$$

Question 3. Light bulbs produced by a manufacturer are known to last 600 hours or more (non-defective) with probability 0.9. A box of 20 bulbs is purchased. What is the probability that at least 2 of the bulbs are defective ?

(a) 0.1216

(b) 0.6083

(c) 0.7300

(d) 0.4115

(e) none of the preceding.

Solution: Let

$$X = \#\text{defective light bulbs..}$$

Then

$$X \sim Binomial(20, 0.1)$$

$$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - 0.9^{20} - 20(0.1)(0.9)^{19} = 0.608253.$$

Question 4. Determine how many different words (meaningful or meaningless) can be created by reordering the letters used in *STATISTICS*.

(a) 3628800

(b) 100

(c) 50400

(d) 66000

(e) 125000

Solution:

$$\frac{10!}{3!3!2!} = 50400.$$

Question 5. There are 10 defective laptops in a shipment of 30 laptops. If we select 5 laptops randomly, what is the probability that among selected laptops only 2 are defective?

(a) 0.98

(b) 0.64

(c) 0.02

(d) 0.45

(e) 0.36

Solution:

$$\frac{\binom{10}{2}\binom{20}{3}}{\binom{30}{5}} = 0.36.$$

Question 6. It is known that 10% of computers of a certain brand are sold defective and 50% of the defective computers will be returned by customers. Assume that 100,000 of these computers are sold. How many of the sold computers were defective *and* will be returned?

(a) 5,000

(b) 50,000

(c) 10,000

(d) 25,000

(e) None of the preceding.

Solution: First, note that $P(\text{Retrun} \mid \text{Defective}) = 0.5$ and $P(\text{Defective}) = 0.1$. Hence,

$$P(\text{Retrun } \cap \text{Defective}) = P(\text{Retrun} \mid \text{Defective}) \times P(\text{Defective}) = (0.1)(0.5) = 0.05.$$

Therefore, 5% of the computers are defective and will be returned. In 100,000 computers, 100,000(0.05)=5000 computers will be returned and are defective. Answer is 5000.

Question 7. Suppose that there are two viruses (V1 and V2) affecting university computers. Both viruses may cause the computers to restart repeatedly (event: R). There is 20% chance that a computer affected by V1 restarts repeatedly and 80% chance that a computer affected by V2 restarts repeatedly. Our estimate is that 70% of computers are restarting repeatedly and 50% of computers are affected by V2. If a computer is restarting repeatedly, determine the probability that it is affected by V2.

(a) 0.57

(b) 0.43

(c) 0.05

(d) 0.99

(e) 0.01

Solution: Define the following events:

Infected with V1= V_1, infected with V2= V_2 and Restarts repeatedly= R. Therefore summarize the question as follows

$$P(R|V_1) = 0.2, P(R|V_2) = 0.8, P(R) = 0.7, P(V_2) = 0.5.$$

Using Bayes' rule

$$P(V_2|R) = \frac{P(R|V_2)P(V_2)}{P(R)} = \frac{(0.8)(0.5)}{0.7} = 0.57.$$

Question 8. A video game is designed such that a player tries to defeat an opponent in three stages, i.e. each game has three stages. The player wins the game if he succeeds in at least 2 stages (out of 3). The chance of defeating the opponent in any stage is 0.7 and the stages are considered independent. Determine the mean and variance of the number of games until a player wins the game.

(a) $\mu = 2.87, \sigma^2 = 2.27$

(b) $\mu = 2.27, \sigma^2 = 2.87$

(c) $\mu = 2.10, \sigma^2 = 0.63$

(d) $\mu = 1.28, \sigma^2 = 0.35$

(e) $\mu = 4.2, \sigma^2 = 0.63$

Solution:

$$P(\text{Winning a game}) = P(\text{Winning at least two stages in a game}) = \binom{3}{2}(0.7)^2(0.3)^1 + \binom{3}{3}(0.7)^3(0.3)^0 = 0.784$$

If X =number of games until a player wins the game. Then X has a geometric distribution with $p = 0.784$, and

$$E(X) = \frac{1}{p} = 1.27551 \approx 1.28$$

and

$$Var(X) = \frac{1-p}{p^2} = \frac{1-0.784}{0.784^2} = 0.3514161 \approx 0.35.$$

Answer is d.

Question 9. In a simulation, the round trip time between two virtual servers follows the following probability density function where measurements are in milliseconds and c is a constant:

$$f(x) = \begin{cases} \frac{c}{x^2} & \text{if } x > 1 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the probability that the round trip time between servers is more than 100 milliseconds.

(a) $c/100$

(b) $c/(100^2)$

(c) 0

(d) $99c/100$

(e) $99c/(100^2)$

Solution:

$$P(X > 100) = \int_{100}^{\infty} \frac{c}{x^2} dx = c/100.$$