Final Exam - Solutions
MAT 2377
Winter 2011

Short Answer Questions

[4] 1.

(a)

$$1 = \sum_{x=3}^{5} f(x) = 2\,c + c + 2\,c = 5\,c \quad \Rightarrow \quad c = \frac{1}{5}.$$

(b)

$$P(X = 4 | X \geq 4) = \frac{P(\{X = 4\} \cap \{X \geq 4\})}{P(X \geq 4)} = \frac{f(4)}{f(4) + f(5)} = \frac{1/5}{3/5} = \frac{1}{3}$$

(c)

$$E[X] = 3\left(\frac{2}{5}\right) + 4\left(\frac{1}{5}\right) + 5\left(\frac{2}{5}\right) = 4.$$

(d)

$$P(X \geq \mu) = P(X \geq 4) = f(4) + f(5) = \frac{3}{5}.$$

[4] 2.

(a) The histogram is approximately symmetric and there is a linear tendency in the normal probability plot. It is reasonable to assume that the tar content is normally distributed.

(b) We want to test $H_0 : \mu = 14$ against $H_1 : \mu > 14$. The observed value of the test statistic is

$$t_0 = \frac{\overline{x} - 14}{s/\sqrt{n}} = \frac{14.112 - 14}{0.0387} = 2.894.$$

The p-value : $P = P(T > 2.894)$, where T has a $t(24)$ distribution. From the table for the t distribution, we obtain $0.0025 < p\text{-value} < 0.005$. We have sufficient evidence to conclude that $\mu > 14$ at $\alpha = 5\%$.

(c) A 95% confidence interval for the μ is

$$\overline{x} \pm t_{0.025,24} \frac{s}{\sqrt{n}} = 14.112 \pm 2.064\,(0.0387) = [14.03, 14.19].$$

[4] 3.

Ignore this question.

[4] 4.

(a) Ignore this question.

(b) We are testing $H_0 : \mu = 4.5$ against $H_1 : \mu \neq 4.5$ at $\alpha = 5\%$. The observed value of the z-test statistic is

$$z_0 = \frac{3.9 - 4.5}{1.5/\sqrt{25}} = -2.$$

The P-value is

$$P = 2\,P(Z > |-2|) = 2\,P(Z > 2.0) = 2\,(1 - 0.97725) = 0.0455.$$

Since $P < \alpha$, we can reject the null hypothesis.

Multiple Choice Questions :

[1] 1. The probability of choosing 2 defective bulbs when choosing randomly without replacement 3 bulbs from a population of 12 bulbs in which 3 are defective is

$$\frac{\binom{3}{2}\binom{9}{1}}{\binom{12}{3}} = 0.122.$$

[1] 2. Solving

$$n \geq \left[\frac{z_{0.05}\sigma}{E}\right]^2 = \left[\frac{(1.645)(\sqrt{0.25})}{0.05}\right]^2 = 270.60.$$

We will select $n = 271$ observations.

[1] 3. Let X be the number of calls in 12 minutes. X has a Poisson distribution with mean $\lambda = 20\,(12/60) = 4$. We want

$$P(X = 0) = e^{-4}\frac{4^0}{0!} = 0.0183.$$

Alternative Solution : Let T be the waiting time (in hours) for a call. T follows an exponential with parameter $\lambda = 20$. We want

$$P(T > 12/60) = 1 - F(12/60) = 1 - [1 - e^{-20(12/60)}] = e^{-4} = 0.0183.$$

[1] 4. Ignore this question.

[1] 5. Let X be the number of checked items to obtain a defective item. X has a geometric distribution with $p = 0.01$. We want

$$P(X \geq 50) = \sum_{x=50}^{\infty} (0.99)^{x-1}\, 0.01 = \frac{(0.99)^{49}(0.01)}{1 - 0.99} = 0.61.$$

[1] 6.

$$P(4 < X < 5) = F(5) - F(4) = \left[1 - \frac{4}{5^2}\right] - \left[1 - \frac{4}{4^2}\right] = 0.09.$$

[1] 7. Let X be a normal random variable with $\mu = 4.35$ and $\sigma = 0.59$. We want

$$P(X > 5.5) = 1 - \Phi\left(\frac{5.5 - 4.35}{0.59}\right) = 1 - \Phi(1.95) = 1 - 0.9744 = 0.0256.$$

[1] 8. The standard error is $0.004/\sqrt{15} = 0.001$.

[1] 9. A 95% confidence interval for μ is

$$\overline{x} \pm t_{0.025,14}\frac{s}{\sqrt{n}} = 0.506 \pm 2.145(0.001) = [0.504, 0.508].$$

[1] 10. Since $\frac{\overline{X}-5}{\sigma/\sqrt{20}} \sim N(0,1)$, then $c = z_{0.1} = 1.28$.

[1] 11. A and B are independent, since

$$P(A)P(B) = (0.8)(0.35) = (0.28) = P(A \cap B).$$

[1] 12. Let X be the number that are ripe and ready to eat among $n = 18$. X has a binomial distribution with $n = 18$ and $p = 0.9$. We want

$$P(X \geq 17) = \binom{18}{17}(0.9)^{17}(0.1)^{1} + \binom{18}{18}(0.9)^{18}(0.1)^{0} = 0.450.$$