MATH 3705A
Test 1 Solutions
January 29, 2015

[Marks]

[4] 1. $\mathcal{L}^{-1}\left\{\dfrac{s+7}{s^2-s-6}\right\} =$

(a) $2e^{-3t}-e^{2t}$ (b) $e^{-3t}-2e^{2t}$ (c) $2e^{3t}+e^{-2t}$ (d) $2e^{3t}-e^{-2t}$ (e) None of these

Answer: (d)

[4] 2. $\mathcal{L}\left\{e^{-2t}\cos(3t)\right\} =$

(a) $\dfrac{s-2}{(s-2)^2+9}$ (b) $\dfrac{s}{(s+2)^2+9}$ (c) $\dfrac{s+2}{(s+2)^2+9}$ (d) $\dfrac{s+2}{s^2+9}$ (e) None of these

Answer: (c)

[4] 3. If $f(t) = \left\{\begin{array}{cc} \sin(2t-6), & t \geq 3 \\ 0, & t<3 \end{array}\right\}$, then $\mathcal{L}\{f(t)\} =$

(a) $\dfrac{e^{-3s}}{s^2+4}$ (b) $\dfrac{2e^{-3s}}{s^2+4}$ (c) $\dfrac{2e^{-6s}}{s^2+4}$ (d) $\dfrac{2e^{-3s}}{s^2+9}$ (e) None of these

Answer: (b)

[4] 4. $\mathcal{L}^{-1}\left\{\dfrac{se^{-3s}}{s^2-2s+5}\right\} =$

(a) $u(t-3)e^{t-3}\left\{\cos[2(t-3)]+\sin[2(t-3)]\right\}$

(b) $u(t-3)e^{t-3}\left\{\cos[2(t-3)]+\dfrac{1}{2}\sin[2(t-3)]\right\}$

(c) $u(t-3)e^{t}\cos(2t)$

(d) $u(t-3)e^{t-3}\cos[2t-6]$

(e) None of the above

Answer: (b)

[7] 5. Employ the Laplace transform to solve the initial-value problem
$y''+2y'-8y=0,\ y(0)=1,\ y'(0)=14.$

Solution:

$[s^2Y(s)-sy(0)-y'(0)]+2[sY(s)-y(0)]-8Y(s)=0 \ \Rightarrow$

$(s^2+2s-8)Y(s)-s-16=0 \ \Rightarrow \ Y(s)=\dfrac{s+16}{s^2+2s-8}=\dfrac{s+16}{(s-2)(s+4)}$

$=\dfrac{3}{s-2}-\dfrac{2}{s+4} \ \Rightarrow \ y(t)=3e^{2t}-2e^{-4t}.$

[7] 6. Employ the Laplace transform to solve the initial-value problem
$y'' + 4y' + 13y = 0,\ y(0) = 3,\ y'(0) = 0.$

Solution:

$[s^2Y(s) - sy(0) - y'(0)] + 4[sY(s) - y(0)] + 13Y(s) = 0 \ \Rightarrow$

$(s^2 + 4s + 13)Y(s) - 3s - 12 = 0 \ \Rightarrow \ Y(s) = \dfrac{3s + 12}{s^2 + 4s + 13} = \dfrac{3(s + 2) + 6}{(s + 2)^2 + 9} \ \Rightarrow$

$y(t) = 3e^{-2t}\cos(3t) + 2e^{-2t}\sin(3t).$