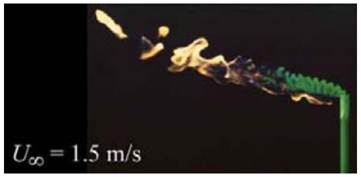
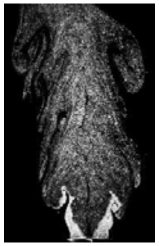
MAAE 2300: Fluid Mechanics I





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Review: Course Outline

3 Major Sections in Course:

- Part I: Introduction (~2 weeks)
- Part II: Fluid Dynamics (~8.5 weeks)
- Part III: Fluid Statics (~1.5 weeks)

Week	Topic
1	**Introduction.** Applications of fluid mechanics. Definitions: fluid, continuum concept, no-slip condition. Fluid properties. Units and dimensions. Pressure and shear stress. Pressure distribution in a fluid in a gravitational field.
2	The atmosphere. Pressure measurement: mercury barometer, manometers. Effect of acceleration on the pressure distribution in a fluid: examples.

Review: Course Outline

Part II: Fluid Dynamics (~8.5 weeks)

Week	Topic
3	**Fluid dynamics.** Streamlines and streamtubes. Steady flow. One-dimensional flow. Control volume approach. Continuity equation.
4	Linear momentum equation for a control volume.
5	Application of linear momentum to steady flows.
6	Euler and Bernoulli equations. Bernoulli equation as an energy equation. Dynamic and total pressure. Applications of Bernoulli: measurement of flow rate (bellmouth inlet, venturi, orifice meter).
7	Angular momentum equation for a control volume. Application to steady flows.
8	Steady-flow energy equation (SFEE). Effects of friction: losses. Concept of hydraulic head. Pump or turbine power; efficiency.
9-10	Viscous flow: laminar versus turbulent flow. Pipe flow analysis: Moody chart. Flow over immersed bodies: boundary layers, entry lengths, minor losses.

Review: Course Outline

Part III: Fluid Statics (~1.5 weeks)

Week	Topic
11	**Fluid statics.** Forces on plane submerged surfaces. Centre of pressure. Examples .
12	Forces on curved submerged surfaces. Examples. Stresses in pipes and vessels due to internal pressure. Forces on submerged bodies: buoyancy. Stability of submerged and floating bodies: centre of buoyancy, metacentre.

Part I: Introduction

Fluid:

- when shear (tangential) force is applied, fluid moves. Fluid continues to move as long as force is applied.
- **Rate of deformation** is proportional to applied force

$$\tau = \mu \frac{du}{dy} \qquad \text{Shear Stress} = \text{Viscosity} \times \underbrace{\text{rate of shear strain}}_{=velocity\ gradient}$$

- Some key concepts (partial list):
  - Continuum concept
  - No-slip condition
  - Viscosity
  - Specific weight
  - Specific gravity
  - Density
  - Ideal gas law
  - Pressure is a scalar
  - Absolute & gauge pressure
  - Boundary layers
  - Surface tension / cavitation

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Part I: Introduction

Basic fluid statics:

- $\therefore \frac{dP}{dy} = -\rho g$ (as derived in class)
- For constant ρg: $\int_1^2 dP = -\rho g \int_1^2 dy$
  $\Delta P = -\rho g \Delta y$
- Or simply: $\Delta P = \rho g \Delta h$ (pressure increases as you move down)

- Pressure measurement devices (e.g. manometers, piezometers, Bourdon tubes, etc.)
- Pressure variation for variable ρg
- Pressure variation in an accelerating fluid *(if g in y-direction)*:

$$\therefore \frac{\partial P}{\partial x} = -\rho a_x \qquad \therefore \frac{\partial P}{\partial y} = -\rho a_y - \rho g \qquad \therefore \frac{\partial P}{\partial z} = -\rho a_z$$

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Part II: Fluid Dynamics

- More basic concepts (partial list):
  - Lagrangian / Eulerian viewpoint
  - Steady and unsteady flow
  - 1D, 2D, 3D flow
  - Pathlines / streaklines / streamlines / streamtubes
  - Laminar / turbulent flow
- Concept of a Control Volume
- Integral equations for a Control Volume:
  - Continuity
  - Linear Momentum
  - Angular Momentum
  - Conservation of Energy
- Specific applications derived from basic equations:
  - Bernoulli equation
  - Euler turbomachinery equation
  - Steady flow Enegy Equation (SFEE)

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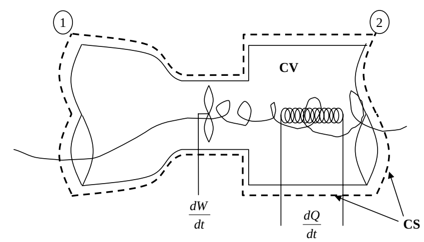
Control Volume Approach



- Basic governing equations in CV form:
  - Continuity
  - Linear Momentum & Angular Momentum
  - Energy Equation (1st Law)

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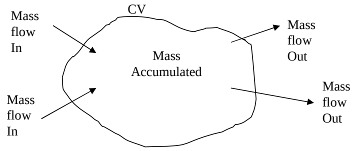
Continuity Equation:

For a CV:



Mass is conserved:

$$\underbrace{(\text{Rate of accumulation of mass in CV})}_{i} = \underbrace{(\text{Rate of mass flow in through CS})}_{ii} - \underbrace{(\text{Rate of mass flow out through CS})}_{iii}$$

$$\underbrace{\frac{d}{dt}\left(\int_{\forall} \rho d\forall\right)}_{\dot{m}_{accumulated}} = \underbrace{\int_{A_{in}} \rho V_n dA}_{\dot{m}_{in}} - \underbrace{\int_{A_{out}} \rho V_n dA}_{\dot{m}_{out}}$$

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Continuity Equation:

If flow is "steady":
By definition of steady flow: $\frac{d}{dt}(...) = 0$

so that $\frac{d}{dt}\left(\int_{\forall} \rho d\forall\right) = 0$ and mass in CV is constant with time.

Continuity Eq. becomes: $\int_{A_{in}} \rho V_n dA = \int_{A_{out}} \rho V_n dA$

$$\therefore \dot{m}_{in} = \dot{m}_{out}$$

If flow is steady and "incompressible" (ρ=constant):

$\rho \int_{A_{in}} V_n dA = \rho \int_{A_{out}} V_n dA$ and thus, $\int_{A_{in}} V_n dA = \int_{A_{out}} V_n dA$

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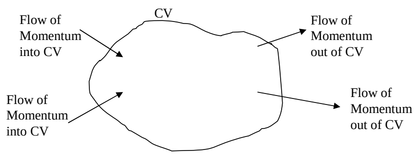
Linear Momentum Equation:



- Mass flow also carries momentum (& energy) into and out of CV
- If fluid leaves, CV with a different amount of momentum than it had when it entered, then a force must have been exerted on the fluid by the CV

Linear Momentum for a CV:

Sum of forces on fluid in CV =

flow rate of momentum out of CV (through CS)

- flow rate of momentum into CV (through CS)

+ rate of accumulation of momentum inside CV

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Linear Momentum Equation:

General form of Linear Momentum Equation:

$$\underbrace{\sum \vec{F}}_{\substack{\text{All forces} \\ \text{(surface + body} \\ \text{forces) on CV}}} = \underbrace{\frac{d}{dt}\int_{CV} \vec{V}\rho d\forall}_{\substack{\text{Change in momentum} \\ \text{within CV}}} + \underbrace{\int_{A_{out}} \vec{V}_{out} \rho V_n dA}_{\text{momentum leaving CV}} - \underbrace{\int_{A_{in}} \vec{V}_{in} \rho V_n dA}_{\text{momentum entering CV}}$$

$$= \frac{d}{dt}\int_{CV} \vec{V}\rho d\forall + \int_{A_{out}} \vec{V}_{out} d\dot{m} - \int_{A_{in}} \vec{V}_{in} d\dot{m}$$

Note:

- This is a vector equation with components in x ,y, z direction:
  $\sum \vec{F} = \sum F_x \hat{i} + \sum F_y \hat{j} + \sum F_z \hat{k}$
  $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$
- These equations are only valid for non-accelerating reference frames.

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Linear Momentum Equation:

Steady Flow Linear Momentum Equations ($\frac{d}{dt}(\ldots)=0$)**:**
In x-direction:

$$\sum \vec{F}_x = \underbrace{\int_{A_{out}} u_{out}\rho V_n dA}_{\text{flow rate of x-wise momentum at outlet(s) of CV}} - \underbrace{\int_{A_{in}} u_{in}\rho V_n dA}_{\text{flow rate of x-wise momentum at inlet(s) of CV}}$$

Similarly in y-direction:
(& also z-direction)

$$\sum \vec{F}_y = \int_{A_{out}} v_{out}\rho V_n dA - \int_{A_{in}} v_{in}\rho V_n dA$$

Important Notes:

- These equations are only valid for steady flow
- These equations are only valid for unaccelerated reference frames.
- For reference frames moving at constant velocity, V_n, must be defined relative to moving reference frame.
- Flow velocities u_{in}, u_{out}, etc. must be defined releative to inertial frame (either relative to constant velocity CV or relative to stationary frame)

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Linear Momentum Equation:

Special case:

- **1D, Steady Flow Linear Momentum Equations:**
  - If 1D, velocity is uniform at any one inlet or outlet and thus does not vary with dA

e.g.:

$$\int_{A_{out}} u_{out}\rho V_n dA = (u\rho V_n)_{out}\int_{A_{out}} dA$$
$$= (u\rho V_n)_{out} A_{out}$$
$$= u_{out}(\rho V_n A)_{out}$$
$$= u_{out}\dot{m}_{out}$$

$\therefore$ For 1D, steady flow, linear momentum equations are:

$$\sum \vec{F}_x = \sum_{outlets}(\dot{m}u)_{out} - \sum_{inlets}(\dot{m}u)_{in}$$
$$\sum \vec{F}_y = \sum_{outlets}(\dot{m}v)_{out} - \sum_{inlets}(\dot{m}v)_{in}$$
$$\sum \vec{F}_z = \sum_{outlets}(\dot{m}w)_{out} - \sum_{inlets}(\dot{m}w)_{in}$$

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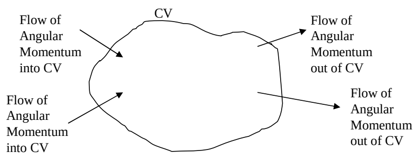
Angular Momentum Equation:



- If fluid leaves, CV with a different amount of Angular momentum than it had when it entered, then a Moment must have been exerted on the fluid by the CV

Angular Momentum for a CV:

Sum of moments on fluid in CV =

flow rate of angular momentum out of CV (through CS)

- flow rate of angular momentum into CV (through CS)

+ rate of accumulation of angular momentum inside CV

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Angular Momentum Equation:

General form of Angular Momentum Equation (*for an inertial ref. frame!*):

$$\sum \vec{M} = \underbrace{\sum \vec{r}\times\vec{F}}_{\text{Moment about origin due to surface + body forces on CV}} = \underbrace{\frac{d}{dt}\int_{CV}(\vec{r}\times\vec{V})\rho d\forall}_{\text{Change in total Ang. momentum within CV}} + \underbrace{\int_{A_{out}}(\vec{r}\times\vec{V}_{out})\rho V_n dA}_{\text{Ang. momentum leaving CV}} - \underbrace{\int_{A_{in}}(\vec{r}\times\vec{V}_{in})\rho V_n dA}_{\text{Ang. momentum entering CV}}$$

For moments X-Y plane (about z-axis) only:

$$\sum \vec{M}_z = \frac{d}{dt}\int_{CV}(\vec{r}\times\vec{V})_z\rho d\forall + \int_{A_{out}}(\vec{r}\times\vec{V}_{out})_z\rho V_n dA - \int_{A_{in}}(\vec{r}\times\vec{V}_{in})_z\rho V_n dA$$

For moments X-Y plane (about z-axis) only and for Steady flow:

$$\sum \vec{M}_z = \int_{A_{out}}(\vec{r}\times\vec{V}_{out})_z\rho V_n dA - \int_{A_{in}}(\vec{r}\times\vec{V}_{in})_z\rho V_n dA$$

For moments X-Y plane (about z-axis) only and for Steady flow but writing in terms of tangential velocity components, V_t:

$$\sum \vec{M}_z = \int_{A_{out}}(\vec{r}V_t)\rho V_n dA - \int_{A_{in}}(\vec{r}V_t)\rho V_n dA$$

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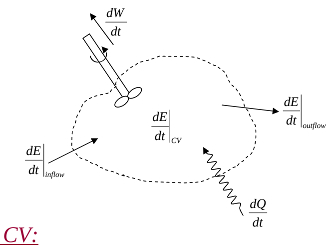
Energy Equation:



1st Law (Conservation of Energy) for a CV:

$$\frac{dQ}{dt}-\frac{dW}{dt}=\left.\frac{dE}{dt}\right|_{CV}+\left.\frac{dE}{dt}\right|_{outflow}-\left.\frac{dE}{dt}\right|_{inflow}$$

where: $E \equiv \underbrace{E_{kinetic}}_{\frac{1}{2}mV^2}+\underbrace{E_{potential}}_{mgh}+\underbrace{E_{internal}}_{\hat{U}\text{ (from thermodynamics energy associated with atomic structure)}}+\underbrace{E_{other}}_{\text{chemical, nuclear, electrostatic or magnetic effects, etc. (NEGLECT)}}$

On a per unit mass basis, $e = e_{kinetic} + e_{potential} + e_{internal}$

where: $e_{kinetic} = \frac{1}{2}V^2$ $\quad e_{internal} = \hat{u}$ (for ideal gas $\hat{u} = c_v T$) $\quad e_{potential} = gz$ (z is up)

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General form of energy equation for a CV:

$$\underbrace{\dot{Q}}_{\text{heat transfer from surroundings}} - \underbrace{\dot{W}_{shaft}}_{\text{"shaft" power from fluid to the rotating shaft}} = \underbrace{\frac{d}{dt}\int_{CV} e\rho d\forall}_{\text{rate of change of energy within CV}} + \underbrace{\int_{out}\left(e+\frac{P}{\rho}\right)\rho V_n dA}_{\text{energy flow out through CS including flow work (power) due to pressure}} - \underbrace{\int_{in}\left(e+\frac{P}{\rho}\right)\rho V_n dA}_{\text{energy flow in through CS including flow work (power) due to pressure}}$$

Where: $e = e_{kinetic} + e_{potential} + e_{internal} = \frac{1}{2}V^2 + gz + \hat{u}$

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Steady flow energy equation (SFEE)

$$\underbrace{\dot{Q}}_{\text{heat transfer from surroundings}} - \underbrace{\dot{W}_{shaft}}_{\text{"shaft" power from fluid to the rotating shaft}} = \underbrace{\int_{out}\left(\tfrac{1}{2}V^2+gz+\hat{u}+\frac{P}{\rho}\right)\rho V_n dA}_{\text{energy flow out through CS including flow work (power) due to pressure}} - \underbrace{\int_{in}\left(\tfrac{1}{2}V^2+gz+\hat{u}+\frac{P}{\rho}\right)\rho V_n dA}_{\text{energy flow in through CS including flow work (power) due to pressure}}$$

Recall from thermodynamics, definition of enthalpy, $\hat{h} \equiv \hat{u} + \frac{P}{\rho}$

$$\underbrace{\dot{Q}}_{\text{heat transfer from surroundings}} - \underbrace{\dot{W}_{shaft}}_{\text{"shaft" power from fluid to the rotating shaft}} = \underbrace{\int_{out}\left(\tfrac{1}{2}V^2+gz+\hat{h}\right)\rho V_n dA}_{\text{energy flow out through CS including flow work (power) due to pressure}} - \underbrace{\int_{in}\left(\tfrac{1}{2}V^2+gz+\hat{h}\right)\rho V_n dA}_{\text{energy flow in through CS including flow work (power) due to pressure}}$$

If 1D flow at inlets and outlets:

$$\dot{Q}-\dot{W}_{shaft} = \dot{m}_{out}\left(\tfrac{1}{2}V^2+gz+\frac{P}{\rho}+\hat{u}\right)_{out} - \dot{m}_{in}\left(\tfrac{1}{2}V^2+gz+\frac{P}{\rho}+\hat{u}\right)_{in}$$

Or in terms of enthalpy: $\dot{Q}-\dot{W}_{shaft} = \dot{m}_{out}\left(\tfrac{1}{2}V^2+gz+\hat{h}\right)_{out} - \dot{m}_{in}\left(\tfrac{1}{2}V^2+gz+\hat{h}\right)_{in}$

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General Procedure for solving CV Problems:

- If possible, choose a frame reference that makes the flow appear steady
  - If unsteady, need to use more general form of linear momentum equation
- Enclose the region of interest <u>completely</u> by a CV
  - Place boundaries where you have information about the flow (i.e. where you know pressure, velocity, etc.) and where you are trying to determine information (e.g. an unknown force, pressure, velocity, etc.)
- Treat CV like a FBD
  - Like a FBD, the CV isolates the region of interest from the surroundings
  - Surroundings are replaced by forces (pressure and shear stresses) and moments they exert <u>on</u> the CV
  - If CV, cuts through a solid component it exposes the internal forces and moments in the solid and these <u>also</u> act on the CV and must be included! (exactly as if CV were a free body)
- Apply equations of motion for the CV
  - Continuity Equation (usually first)
  - Momentum equations Second
  - Be careful to correctly apply signs of velocities and forces relative to assumed directions
- Check calculated results
  - Mentally check calculated values and directions especially with respect to signs and assumed directions

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Applications of Governing Equations:

- **Bernoulli Equation**:
  - Derived by writing linear momentum equation along an arbitrary streamline to get Euler equation, and then integrating that equation along the streamline assuming steady, incompressible flow:

Bernoulli Equation: $P + \frac{1}{2}\rho V^2 + \rho g z = \text{constant}$

Limitations:
- Steady flow
- Incompressible flow (valid for most liquids and gases as well if Ma < 0.3)
- **Frictionless (inviscid) flow** (very restrictive assumption!!)
- Valid only along a streamline (can later be shown to be valid for irrotational flow)
- No heat transfer or work

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Bernoulli Equation

$P + \frac{1}{2}\rho V^2 + \rho g z = \text{constant}$

Static, Dynamic,& Stagnation Pressures

- $P \equiv$ pressure fluid actually exerts on surroundings
  - Known as static pressure;
  - Sometimes written as P_s
  - Pressure one would "feel" when moving along with fluid particles
- ½ ρV^2 known as "dynamic pressure"
  - Equivalent Pressure associated with the velocity of the fluid
- $P + ½\, \rho V^2 \equiv P_0 \equiv$ "total pressure" or "stagnation pressure"
  - Pressure which would occur if the fluid was brought to rest (stagnation)
  - E.g. when fluid impacts a body

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Applications of Bernoulli Equation

- Flow measurment
  - Pitot tubes / Pitotstatic tubes
  - Venturi meter, orifice plate, bellmouth inlet etc.
- Interpreting flows
  - Curved streamlines vs. straight streamlines

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Application of SFEE

Rewrite SFEE:

$$\underbrace{\frac{P_1}{\rho g}}_{\text{static head, } h_s} + \underbrace{\frac{V_1^2}{2g}}_{\text{dynamic head, } h_v} + \underbrace{z_1}_{\text{elevation head, } h_z} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \underbrace{\frac{\dot{W}_{shaft}}{\dot{m} g}}_{\text{head supplied by pump, } h_p \text{ (negative) or extracted by turbine, } h_t \text{ (positive)}} + \underbrace{\left(\frac{\hat{u}_2 - \hat{u}_1}{g} - \frac{\dot{Q}}{\dot{m} g}\right)}_{\text{"head loss" due to friction and heat transfer, } h_f}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_{pump} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_{turbine} + h_{friction}$$

Total head: $H = \frac{P}{\rho g} + \frac{V^2}{2g} + z$

Pump Head: $h_p = \frac{\dot{W}_{shaft}}{\dot{m} g}$

Turbine Head: $h_t = \frac{\dot{W}_{shaft}}{\dot{m} g}$

Hydraulic power:
$\dot{W}_h = \dot{m} g h = \rho g Q h$

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Application of SFEE

Pump & Turbine Efficiency:

Pump Efficiency:

$$\eta_p = \frac{\text{Hydraulic Power}}{\text{Mechanical Input Power}} = \frac{\dot{W}_h}{\dot{W}_m} = \frac{\rho Q g h_p}{\omega T}$$

Turbine Efficiency:

$$\eta_t = \frac{\text{Mechanical Power Output}}{\text{Hydraulic Power Input}} = \frac{\dot{W}_m}{\dot{W}_h} = \frac{\omega T}{\rho Q g h_t}$$

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Frictional Losses in Pipes & Ducts

- For **<u>Fully Developed</u>** flow:
  - From derivation using linear momentum eq.:

$$h_f = \frac{\tau_w}{\rho g}\frac{4L}{D}$$

  - We then defined Darcy-Weisbach friction factor, f

$$f = \frac{4\tau_w}{\frac{1}{2}\rho V^2}$$

  - So finally:

$$\boxed{h_f = f\,\frac{L}{D}\frac{V^2}{2g}}$$

$$f = f\left(\text{Re}, \frac{\varepsilon}{D}, \text{pipe cross-sectional shape}\right)$$

$$\text{Roughness} = \frac{\varepsilon}{D} = \frac{\text{roughness height}}{\textit{diameter}}$$

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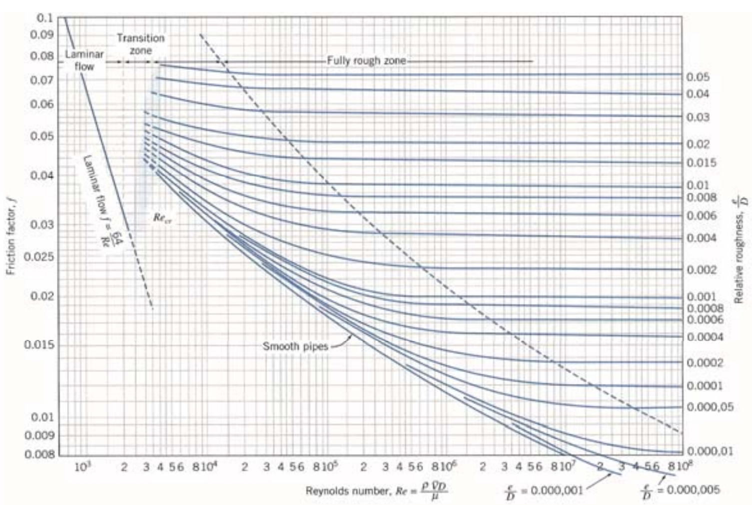
Moody Diagram



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Friction factor, f, and pipe flow losses

Laminar Flow:

- From solution to linear momentum equation:

$$f = \frac{64}{\text{Re}}$$

  - for circular pipes (independent of roughness as long as still laminar!)
  - $f_{laminar}$ is inversely proportional to velocity
  - ∴ head loss in laminar flow is **linearly** proportional to flow velocity and not velocity squared as in turbulent flow

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Friction factor, f, and pipe flow losses

Turbulent Flow

- Colebrook (1939):

$$\frac{1}{f^{0.5}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

  - Most widely used expression
  - Implicit (must iterate) but trivial on programmable calculator or computer
  - Moody diagram is a log-log plot of this correlation in turbulent regime

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"Minor Losses" in Piping Systems

- Losses are **in addition to** head loss for same length of straight pipe and vary primarily with the dynamic head or dynamic pressure of the flow

$$\Delta h_m = \sum K \frac{V^2}{2g}$$

where K is the minor loss coefficient (determined from experiment and given in charts & tables)

- Thus total losses for system ("minor losses" + frictional losses in piping) may be expressed as:

$$\Delta h_f = f\,\frac{L}{D}\frac{V^2}{2g} + \sum K \frac{V^2}{2g}$$

$$\Delta h_f = \left(\sum K + f\,\frac{L}{D}\right)\frac{V^2}{2g}$$

- <u>**Note must sum each section of pipe separately if V changes!**</u>

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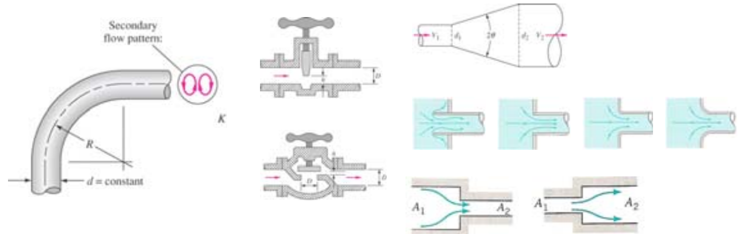
"Minor Losses" in Piping Systems

- Head losses due to fittings, entries, exits, valves, bends, etc.
- Traditionally called "minor losses" but often they are a major portion of the total head loss in a system
- Physically represent additional energy dissipation due to induced secondary flows, separation, recirculation etc.



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Empirical Data for Minor Loss Coefficient, K

TABLE 1–1 Loss Coefficients for Fittings and Valves

Type of fitting or valve	Loss coefficient (K)
45° elbow, standard	0.35
45° elbow, long radius	0.2
90° elbow, standard	0.75
Long radius	0.45
Square or miter	1.3
180° bend, close return	1.5
Tee, standard, along run, branch blanked off	0.4
Used as elbow, entering run	1.3
Used as elbow, entering branch	1.5
Branching flow	1.0
Coupling	0.04
Union	0.04
Gate valve, open	0.17
¾ open	0.9
½ open	4.5
¼ open	24.0
Diaphragm valve, open	2.3
¾ open	2.6
½ open	4.3
¼ open	21.0
Globe valve, bevel seat, open	6.4
½ open	9.5
Composition seat, open	6.0
½ open	8.5
Plug disk, open	9.0
¾ open	13.0
½ open	36.0
¼ open	112.0
Angle valve, open	3.0
Y or blowoff valve, open	3.0
Plug cocks, θ = 5°	0.05
10°	0.29
20°	1.56
40°	17.3
60°	206.0
Butterfly valve, θ = 5°	0.24
10°	0.52
20°	1.54
40°	10.8
60°	118.0
Check valve, swing	2.0
Disk	10.0
Ball	70.0
Foot valve	15.0
Water meter, disk	7.0
Piston	15.0
Rotary (star-shaped disk)	10.0
Turbine wheel	6.0

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Empirical Data for Minor Loss Coefficient, K

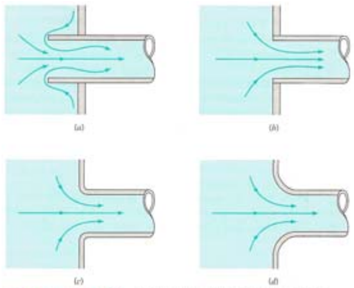
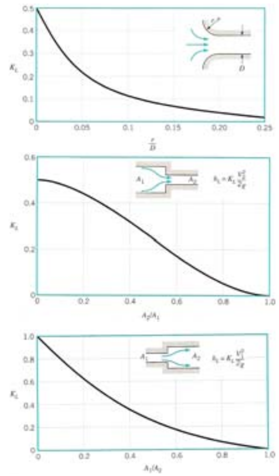


FIGURE 8.22 Entrance flow conditions and loss coefficient (Refs. 28, 29). (*a*) Reentrant, $K_L = 0.8$, (*b*) sharp-edged, $K_L = 0.5$, (*c*) slightly rounded, $K_L = 0.2$ (see Fig. 8.24), (*d*) well-rounded, $K_L = 0.04$ (see Fig. 8.24).



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Pipe Roughness Values

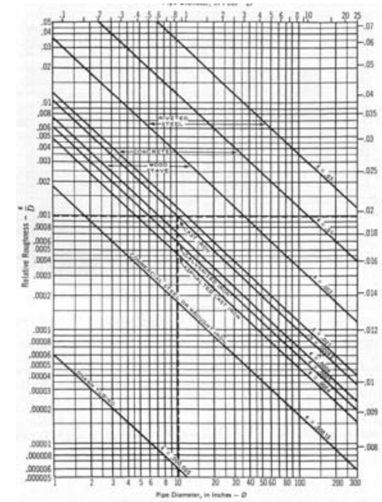

FIGURE 1-5 Relative roughness of pipe material. (Used with permission of ASME, from L. F. Moody, "Friction Factors for Pipe Flow." *ASME Transactions*, 1944.)

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Viscous Flow in Pipes & Ducts

- Laminar vs. turbulent flow
  - Reynolds number
    - Ratio of mometum and viscous forces

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{4Q}{\pi \nu D} = \frac{4\dot{m}}{\pi \mu D}$$

  - Entry lengths for flow to become fully developed
- For turbulent flow in non-circular ducts:
  - Define and use "Hydraulic diameter", D_h:

$$D_h \equiv \frac{4A}{Perimeter} = \frac{4 \times \text{cross-sectional area}}{\text{wetted perimeter of pipe or duct}}$$

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Solving Viscous Pipe Flow Problems

Type I: Head loss Problem:

- Find H for known Q, ρ, μ, D, L (known piping & fluid):
- No iteration required
  - From Q and piping info, find Re, $\frac{\varepsilon}{D}$, and f
  - Calculate head loss as $\Delta h_f = \left(\sum K + f\frac{L}{D}\right)\frac{V^2}{2g}$
  - Plug into SFEE equation to find required pump head

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Solving Viscous Pipe Flow Problems

Type II: Flow rate problem :

- Find Q for known H, ρ, μ, D, L (known piping & fluid):
- Requires iteration
  - Use SFEE to find allowable head loss
  - Guess a friction factor (0.02 is often a good starting point if flow is turbulent)
    - Calculate V from $\Delta h_f = \left(\sum K + f\frac{L}{D}\right)\frac{V^2}{2g}$
    - Now use V to find Re, and use $\frac{\varepsilon}{D}$ to find new f
  - Iterate until f converges
  - Q = VA

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Solving Viscous Pipe Flow Problems

Type III: Pipe sizing problem:

- Find D for known H, Q, ρ, μ, L
- Requires iteration
  - More difficult since D appears in all terms: Re, $\frac{\varepsilon}{D}$, $\Delta h_f = \left(\sum K + f\frac{L}{D}\right)\frac{V^2}{2g}$
  - Different procedures can be used for iteration depending on preference and whether Q or V is known initially
  - E.g. if Q is known, use SFEE to solve for h_f, and rewrite h_f & Re in terms of Q:
    - Guess f
    - Solve for diameter in terms of friction factor using eq. based on:

$$\Delta h_f = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D2g}\left(\frac{4Q}{\pi D^2}\right)^2 = \frac{8fLQ^2}{\pi^2 gD^5}$$

    - Solve for Re & roughness

$$\mathrm{Re} = \frac{4Q}{\pi D\nu} \qquad \frac{\varepsilon}{D}$$

    - Find new f and iterate until converged (fairly quick)

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Solving Viscous Pipe Flow Problems

Type III: Pipe sizing problem (Alternate approach):

- Find D for known H, Q, ρ, μ, L
- Since commercial pipe only comes in standard sizes, you might only be interested in solutions for specific D
- Procedure:
  - Guess D (i.e. choose best guess from available D)
  - Solve a Type II unknown flow problem for each guessed D
    - See type II procedure on previous slide
  - Select D that gives best solution for your constraints (capital cost, operating cost, time, availability, etc.)

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Part III: Fluid Statics

- See summary notes on Fluid Statics uploaded separately…

Good luck on the Exam and enjoy your summer!

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