DGD (circle one):
DGD1 (in DMS1120)
DGD2 (in STEB0138)

Last name: Answers
First name:
Student number:

Marks: /11

MAT 1348B — Third Homework Assignment: ANSWERS

Instructions: **Print this document to hand in**. *Show all relevant work to receive full credit. Submit a finished product, not a draft. You may write on both sides of the paper or insert additional pages if necessary. Please staple the pages. Submit the assignment to your TA in the DGD or in the appropriate submission box in the Department of Mathematics and Statistics. Late assignments will not be accepted.*

1. Prove that the argument below is **valid** using the **rules of inference** notes on the web (Essentially Rosen, p. 72), as well as Boolean Equivalences. *Name the rule of inference or equivalence used at each step.* [4pts]

$$\begin{array}{c} b \to c \\ \neg(c \wedge \neg a) \\ a \vee b \\ \hline \therefore a \end{array}$$

Solution. Here is one solution. There can be many valid answers.

1.	$b \to c$	Hyp
2.	$\neg(c \wedge \neg a)$	Hyp
3.	$a \vee b$	Hyp
4.	$\neg c \vee \neg\neg a$	De Morgan, 2.
5.	$\neg c \vee a$	$\neg\neg$ – Law, 4
6.	$\neg b \vee c$	Boolean Equiv, 1
7.	$c \vee \neg b$	Boolean Equiv (commutativity of $\vee$), 6
8.	$\neg b \vee a$	Cut (or Resolution), 7, 5
9.	$b \vee a$	Boolean Equiv (commutativity of $\vee$), 3
10.	$a \vee a$	Cut (or Resolution), 9, 8
11.	a	Boolean Equiv (idempotent laws for $\vee$), 10

2. Let P_1, P_2, P_3, and C be four propositions, and consider the argument

$$\begin{array}{c} P_1 \\ P_2 \\ P_3 \\ \hline \therefore C \end{array}$$

Assuming that the set $\{P_1, P_2, P_3\}$ of the premises (or hypotheses) is **inconsistent**, what can you say about the validity of the argument? *Briefly justify your answer.* [2pts]

SOLUTION. The argument is valid. The reason is that the argument above is valid if and only if

$$((P_1 \wedge P_2 \wedge P_3) \to C)$$

is a tautology. But if the premises are inconsistent, then $(P_1 \wedge P_2 \wedge P_3) \equiv \mathbf{F}$, and we know $\mathbf{F} \to C$ is a tautology (i.e. is always true).

<u>Remarks</u>

Recall, as shown in class, an argument $\begin{array}{c} H_1 \\ \vdots \\ H_n \\ \hline \therefore C \end{array}$ is valid iff

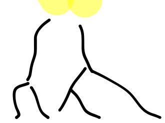
the truth tree with root $\left\{ \begin{array}{l} H_1 \\ \vdots \\ H_n \\ \neg C \end{array} \right.$ has every branch closed.

(this is because the formula $(H_1 \wedge \cdots \wedge H_n) \to C$ must be a tautology; hence, put $\neg[(H_1 \wedge \cdots \wedge H_n) \longrightarrow C]$ at the top of the tree and start the tree. This gives (in the first steps)

$$\begin{array}{c} H_1 \wedge \cdots \wedge H_n \\ | \\ \neg C \\ | \\ \left.\begin{array}{c} H_1 \\ \vdots \\ H_n \end{array}\right\} \text{using the } \wedge\text{-rules repeatedly.} \end{array}$$

Hence, to save effort, it is clearly equivalent to start the truth tree as follows:

1. H_1
$\vdots$
n. H_n
n-1. $\neg C$



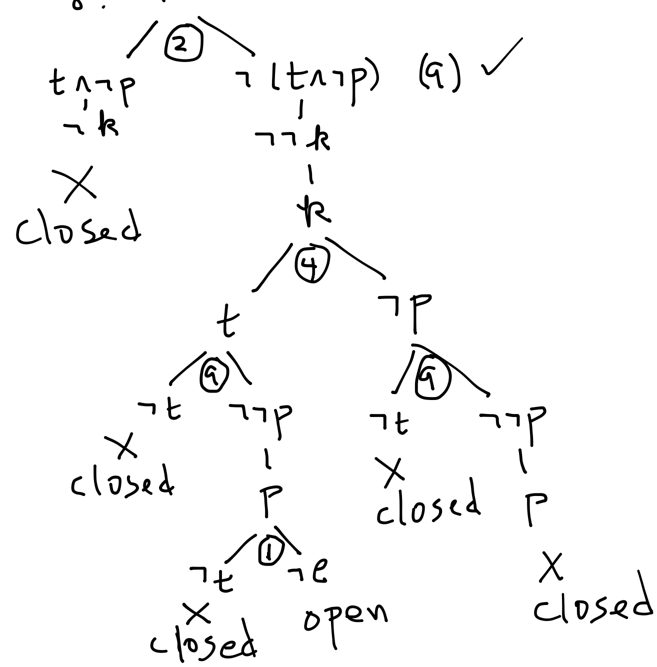
Draw the full tree. If every branch closes, then the argument is valid.

3. Determine whether or not the argument below is **valid** using a **truth tree**. If you claim the argument is not valid, **give all valuations falsifying the argument**, i.e. give all truth-assignments making the argument invalid. [5pts]

$$\begin{array}{c} t \to \neg e \\ (t \wedge \neg p) \leftrightarrow \neg k \\ \neg(k \to e) \\ t \vee \neg p \\ \hline \therefore \neg t \end{array}$$

See the discussion on the previous page. Put the hypotheses and ¬Conclusion at the top of the tree:

1. $t \to \neg e$ ✓
2. $(t \wedge \neg p) \leftrightarrow \neg k$ ✓
3. $\neg(k \to e)$ ✓
4. $t \vee \neg p$ ✓
5. $\neg\neg t$ ✓
6. t } ¬¬-rule, from 5.
7. k } ¬ → -rule, from 3.
8. $\neg e$



On the open branch we obtain a valuation falsifying the argument: $V(t) = V(k) = V(p) =$ True.
$V(e) =$ False.

This valuation V (i.e. truth-assignment) makes all the hypotheses true and the conclusion false. ∴ The argument is not valid.