

DGD (circle one):
 DGD1 (in DMS1120)
 DGD2 (in STEB0138)

Last name: Answers
 First name:
 Student number:

Marks: /15

MAT 1348B — Second Homework Assignment (corrected)— Due Jan. 28, 2015 at 3:30pm

Instructions: Print out this paper and write your answers on it. Make sure to mark which DGD you are in. Show all relevant work to receive full credit. Write neatly. You may write on both sides of the paper or insert additional pages if necessary. Please staple the pages. Submit the assignment to your TA in your DGD or in the appropriate submission box in the Department of Mathematics and Statistics. Late assignments will not be accepted.

1. The table below is a truth table for a mystery formula φ of propositional calculus, containing atoms (propositional variables) p , q , and r , and logical connectives. [6pts]

p	q	r	φ
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

$\leftarrow p \wedge \neg q \wedge r = C_1$
 $\leftarrow \neg p \wedge q \wedge r = C_2$
 $\leftarrow \neg p \wedge \neg q \wedge r = C_3$

- (a) Write a **DNF formula** (Disjunctive Normal Form) for φ .
 (b) Using the **Boolean Algebra equivalences**, show that $\varphi \equiv \neg(p \wedge q) \wedge r$. (See my proofs at the bottom of the handout sheet on Boolean Equivalences.)
 (c) Find a formula of propositional calculus ψ logically equivalent to φ (i.e. $\psi \equiv \varphi$) such that ψ contains **only the logical connectives \neg and \vee** .

Answers: (a) A DNF is $C_1 \vee C_2 \vee C_3$, for C_i above.

(b) Using left associativity,

$$C_1 \vee C_2 \vee C_3 \equiv (p \wedge \neg q) \wedge r \vee (\neg p \wedge q) \wedge r \vee ((\neg p \wedge \neg q) \wedge r) \quad (*)$$

Notice by commutativity, left distributivity \equiv right distributivity

e.g. $(a \vee b) \wedge c \equiv c \wedge (a \vee b) \equiv (c \wedge a) \vee (c \wedge b) \equiv (a \wedge c) \vee (b \wedge c)$

$\therefore (*) \equiv ((p \wedge \neg q) \wedge r) \vee ((\neg p \wedge r) \wedge q) \vee ((\neg p \wedge r) \wedge \neg q)$, by comm. & assoc. several times

$\equiv ((p \wedge \neg q) \wedge r) \vee ((\neg p \wedge r) \wedge (q \vee \neg q))$ distrib. (cont'd, next page)

Answers: (a) A DNF is $C_1 \vee C_2 \vee C_3$, for C_i above
 (b) using left associativity,

$$\textcircled{*} C_1 \vee C_2 \vee C_3 \equiv ((p \wedge q) \wedge r) \vee ((\neg p \wedge q) \wedge r) \vee ((\neg p \wedge \neg q) \wedge r)$$

Notice by commutativity, left dist. \equiv right distrib.,
 e.g. $(a \vee b) \wedge c \equiv c \wedge (a \vee b) \equiv (c \wedge a) \vee (c \wedge b) \equiv (a \wedge c) \vee (b \wedge c)$.

$$\therefore \textcircled{*} \equiv ((p \wedge q) \wedge r) \vee ((\neg p \wedge r) \wedge q) \vee ((\neg p \wedge r) \wedge \neg q),$$

by comm. & assoc., several times

$$\equiv ((p \wedge q) \wedge r) \vee ((\neg p \wedge r) \wedge (q \vee \neg q)) \text{ distrib.}$$

$$\equiv ((p \wedge q) \wedge r) \vee ((\neg p \wedge r) \wedge T) \quad \left. \begin{array}{l} \text{Negation Laws} \\ \text{\& Unit Laws} \end{array} \right\}$$

$$\equiv ((p \wedge q) \vee \neg p) \wedge r \quad \text{right distrib. law}$$

$$\equiv ((p \vee \neg p) \wedge (q \vee \neg p)) \wedge r \quad \text{" " "}$$

$$\equiv (T \wedge (q \vee \neg p)) \wedge r \quad \text{Negation law}$$

$$\equiv (q \vee \neg p) \wedge r \quad \text{Unit law}$$

$$\equiv \neg (q \wedge p) \wedge r \quad \text{De Morgan}$$

$$(c) \neg (q \wedge p) \wedge r \equiv \neg (q \wedge p) \vee \neg r$$

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and $\neg \neg$

$$\equiv \neg (\neg (q \vee \neg p) \vee \neg r)$$

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2. Use **truth tables** to answer this question. Consider the following two formulas (compound propositions) in propositional variables (atoms) p, q, r :

[5pts]

$$P: (p \rightarrow r) \vee (q \vee r)$$

$$Q: \neg(p \rightarrow q) \rightarrow r$$

- (a) For each of P and Q , give all truth assignments for which the proposition is **false**.
- (b) For each of P and Q , determine whether it is a tautology, a contradiction (i.e. always false), or neither of the two.
- (c) Are P and Q **logically equivalent**, i.e. is $P \equiv Q$? Explain your answer.

p	q	r	$p \rightarrow r$	$q \vee r$	$\neg(p \rightarrow q)$	P	Q
T	T	T	T	T	F	T	T
T	T	F	F	T	F	T	T
T	F	T	T	T	T	T	T
T	F	F	F	F	T	F	F
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	F	F	F	F	F

- (a) Just $v(p) = T, v(q) = v(r) = F$ makes P, Q false.
- (b) They are both neither tautologies nor a contradiction.
- (c) Yes $P \equiv Q$: their values are identical.

3. Let R be the propositional formula (compound proposition)

$$(((p \vee q) \wedge (p \rightarrow r)) \wedge (q \rightarrow r)) \rightarrow r.$$

Using **truth trees**, determine whether or not R is a **tautology**. If you claim that it is not, give **all counterexamples** (i.e. all valuations making the formula false). [4pts]

$$(1) \quad \neg \left[\left((p \vee q) \wedge (p \rightarrow r) \right) \wedge (q \rightarrow r) \right] \rightarrow r \quad \checkmark$$

$$(2) \quad \checkmark \left((p \vee q) \wedge (p \rightarrow r) \right) \wedge (q \rightarrow r) \quad \left. \vphantom{\left((p \vee q) \wedge (p \rightarrow r) \right) \wedge (q \rightarrow r)} \right\} \text{from (1)}$$

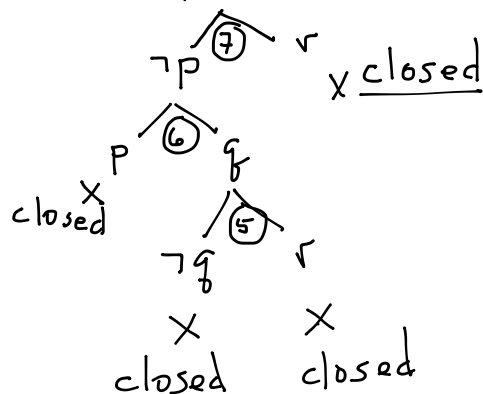
$$(3) \quad \neg r$$

$$(4) \quad \checkmark (p \vee q) \wedge (p \rightarrow r) \quad \left. \vphantom{(p \vee q) \wedge (p \rightarrow r)} \right\} \text{from (2)}$$

$$(5) \quad \checkmark q \rightarrow r$$

$$(6) \quad \checkmark (p \vee q) \quad \left. \vphantom{(p \vee q)} \right\} \text{from (4)}$$

$$(7) \quad \checkmark (p \rightarrow r)$$



Every branch for $\neg R$ closes. $\therefore R$ is a tautology.