

DGD (circle one):

Last name: **ANSWERS**

DGD1 (in DMS1120 )

First name:

DGD2 (in STEB0138 )

Student number:

Marks: /16

## MAT 1348B — First Homework Assignment — ANSWERS

*Instructions: Print out this paper and write your answers on it. Make sure to mark which DGD you are in. Show all relevant work to receive full credit. Write neatly. You may write on both sides of the paper or insert additional pages if necessary. Please staple the pages. Submit the assignment to your TA in your DGD or in the appropriate submission box in the Department of Mathematics and Statistics. Late assignments will not be accepted.*

1. Translate each of the following statements into a Propositional Logic formula (called “Compound Formula” in Rosen), using correct logical connectives. **Define at the bottom of this page the meaning of all propositional variables (atoms) you use in your translation.**

[5pts]

- (a) Passing the course is a sufficient condition for students to do well on the final exam.

*Answer:*  $P \rightarrow W$

- (b) Students do well on the final exam only if they do not miss the midterm exam.

*Answer:*  $W \rightarrow \neg M$

- (c) Not missing the midterm exam and doing the homework is necessary for passing the course.

*Answer:*  $P \rightarrow (\neg M \wedge H)$

- (d) Students do not do well on the final exam if and only if they do not do their homework or miss the midterm exam.

*Answer:*  $\neg W \leftrightarrow (\neg H \vee M)$

- (e) Students do well on the final exam unless they miss the midterm exam.

*Answer:*  $W \vee M$  (equivalently,  $\neg W \rightarrow M$  , equivalently,  $\neg M \rightarrow W$ ).

*Define your propositional variables (atoms) used above:*

*P:* Students pass the course.

*W:* Students do well on the final exam.

*H:* Students do their homework.

*M:* Students miss the midterm exam.

2. Recall, a set of formulas  $\{A_1, \dots, A_n\}$  is consistent if they are simultaneously satisfiable, i.e. if the conjunction of the formulas  $A_1 \wedge \dots \wedge A_n$  is satisfiable. Use a **truth table** to determine whether the given set of four propositions below is **consistent**. Clearly explain how the truth table supports your answer. [5pts]

$$\{ (p \vee (\neg q \vee r)), ((r \leftrightarrow q) \rightarrow p), (\neg r \wedge (p \rightarrow q)), (p \wedge \neg r) \}$$

$p$	$q$	$r$	$\neg q$	$\neg q \vee r$	$(p \vee (\neg q \vee r))$	$r \leftrightarrow q$	$(r \leftrightarrow q) \rightarrow p$	$\neg r$	$p \rightarrow q$	$(\neg r \wedge (p \rightarrow q))$	$(p \wedge \neg r)$
<b>T</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>F</b>	<b>F</b>
<b>T</b>	<b>T</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>
<b>T</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>
<b>T</b>	<b>F</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>F</b>	<b>T</b>
<b>F</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>F</b>	<b>T</b>	<b>F</b>	<b>F</b>
<b>F</b>	<b>T</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>F</b>
<b>F</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>F</b>	<b>F</b>
<b>F</b>	<b>F</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>F</b>	<b>T</b>	<b>T</b>	<b>T</b>	<b>F</b>

Notice, the formulas are all simultaneously true in Row 2 of the truth table; this gives a valuation (= truth-assignment) of values to the atoms making all formulas in the set simultaneously true.

3. On the Island of Knights and Knaves you meet two natives, A and B. Given their statements below, what can you determine about their identities? Fully explain your reasoning.

[6pts]

- (a) A says: "B is a knight but I am a knave". What do you conclude?

*Ans:* A says the conjunction ("B is a knight"  $\wedge$  "A is a knave").

Case 1. Suppose A is a knight. Then A speaks **T**. But A speaks a conjunction, so each conjunct is **T**. In particular, what A says implies he is a knave, which is false. Contradiction.

Case 2. Suppose A is a knave. Then A speaks **F**. But "A is a knave" is **T**. Hence, for the entire statement of A to be **F**, it must be that "B is a knight" is **F**, so B must be a knave. CONSISTENT!

Hence: A, B are both knaves is the only consistent answer.

- (b) B says: "I am a knight only if A is a knight". What do you conclude?

*Ans:* B says ("B is a knight"  $\rightarrow$  "A is a knight"). There are two cases:

Case 1: B is a knave. Therefore he speaks **F**. Therefore the value of ("B is a knight"  $\rightarrow$  "A is a knight") is **F**. Hence the hypothesis is **T** and the conclusion is **F**. So "B is a knight" must be **T**. Contradiction!

Case 2. B is a knight. Then B speaks **T**. Therefore the hypothesis of what B says is **T**, hence the conclusion is **T**, since **T**  $\rightarrow$  **T** = **T**. Hence A is a knight. CONSISTENT !

Hence: A, B are both knights

- (c) In Case (b) above, what do you conclude if A adds: "B is a knave and so am I."?

Now there are two statements:

B says ("B is a knight"  $\rightarrow$  "A is a knight").

A says ("B is a knave"  $\wedge$  "A is a knave").

This is clearly impossible. In the first case, we know from B's answer (in part (b)) that A, B must both be knights (that's the only consistent answer). But if A is a knight, he speaks **T**. But that is impossible: since then (according to A's statement) A, B would both be knaves!

Hence: the above situation cannot happen.