1. (a) [4 pts] Why is $\int_0^6 \frac{5x}{-x^2+4}\,dx$ an improper integral?
If it is convergent, compute its value. If not, explain why it is divergent.

It is improper because $\frac{5x}{-x^2+4}$ is NOT defined AT $x=2$ IN $[0,6]$.

NOTE: $$\int_0^2 \frac{5x}{-x^2+4}\,dx = \lim_{t\to 2^-} 5\cdot\int_0^t \frac{x}{-x^2+4}\,dx = \lim_{t\to 2^-} 5\cdot\int_4^{-t^2+4} \frac{1}{-2\cdot u}\,du =$$

$$= \lim_{t\to 2^-} \frac{5}{-2}\cdot \ln|u|\Big|_4^{-t^2+4} =$$

$$= -\frac{5}{2}\cdot\lim_{t\to 2^-}\left(\ln|-t^2+4| - \ln 4\right) = \infty$$

$u=-x^2+4$; $\frac{du}{dx}=-2x$

$\frac{du}{-2} = x\,dx$

$x=0 \to u=4$

$x=t \to u=-t^2+4$

CONCLUSION: our Integral is Divergent.

(b) [4 pts] Determine if $\int_1^\infty \frac{1}{20\sqrt{x}+x^4}\,dx$ is convergent or divergent using an appropriate comparison test.

~~NOTE that: for all $x \geq 1$ one has $\sqrt{x} \leq x^4$, thus:~~

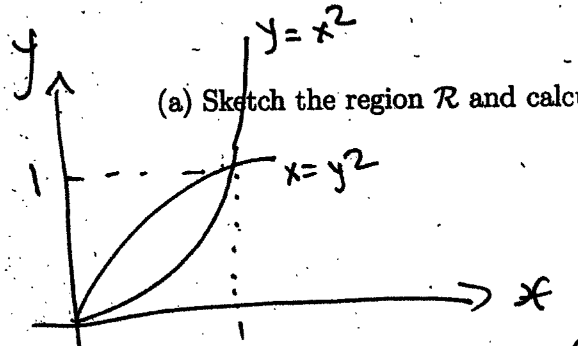
$\boxed{\frac{1}{20\sqrt{x}+x^4} \leq \frac{1}{x^4} \quad \text{FOR ALL } x \geq 1}$

Since $\int_1^\infty \frac{1}{x^4}\,dx$ is Convergent ($p=4>1$) one has by the Comparison Test that our Integral is Convergent

2. [10 pts] Let $\mathcal{R}$ be the region in the first quadrant bounded by the curves

$$y = x^2, x = y^2.$$

(a) Sketch the region $\mathcal{R}$ and calculate its area.



Cut POINTS:

$x = y^2 = (x^2)^2 = x^4 \to x = x^4 \Rightarrow$

$x - x^4 = 0;\ x(1-x^3) = 0$

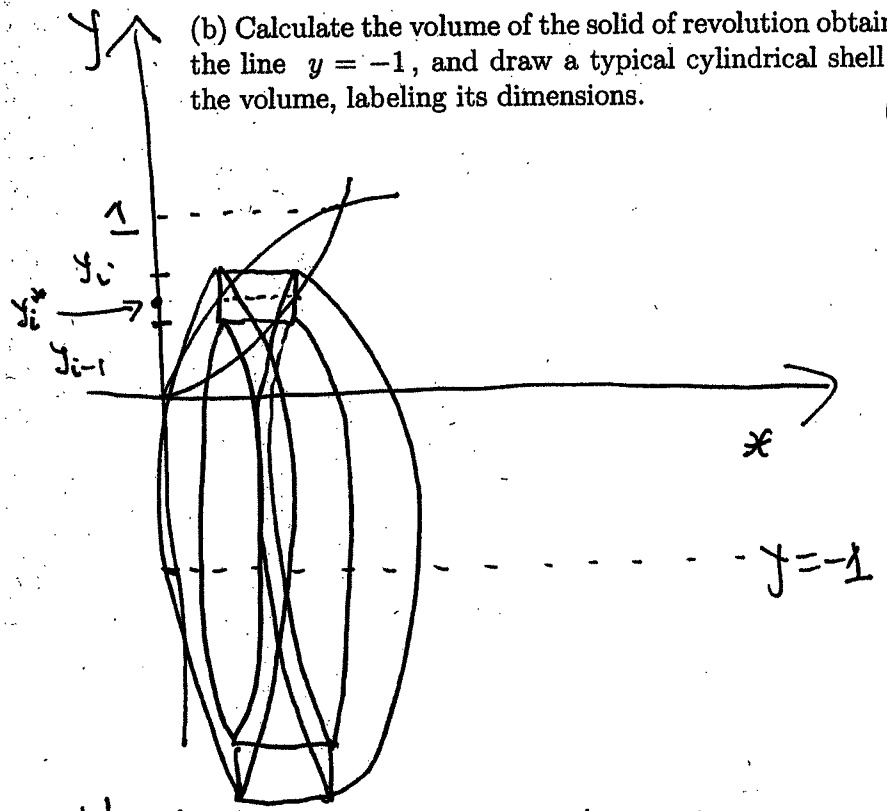
$x(1-x)(1+x+x^2) = 0$

$\begin{cases} x=0 \to y=0 \\ x=1 \to y=1 \end{cases}$

$$A = \int_0^1 (x^{\frac{1}{2}} - x^2)dx = \left(\frac{x^{3/2}}{\frac{3}{2}} - \frac{x^3}{3}\right)\Bigg|_0^1$$

$$= \frac{1}{\frac{3}{2}} - \frac{1}{3} = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

(b) Calculate the volume of the solid of revolution obtained by rotating the region $\mathcal{R}$ around the line $y = -1$, and draw a typical cylindrical shell which appears in the calculation of the volume, labeling its dimensions.



Cut $[0,1]$ into n subintervals of equal length: $\Delta y = \frac{1-0}{n}$, by: $0 = y_0 < y_1 < \dots < y_i < \dots < y_n = 1$

Pick y_i^*: the mid point of $[y_{i-1}, y_i$

V_i = volume of the ith shell

$V_i = C \cdot H \cdot T =$

$= 2\pi(1 + y_i^*)\cdot(\sqrt{y_i^*} - {y_i^*}^2)\cdot$

$\cdot \Delta y$

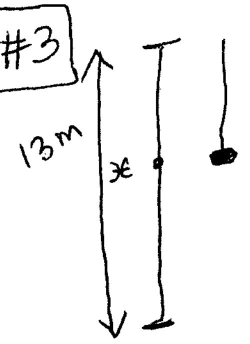
Hence

$$V = \int_0^1 2\pi(1+y)(y^{\frac{1}{2}} - y^2)dy = 2\pi\int_0^1 y^{\frac{1}{2}} - y^2 + y^{\frac{3}{2}} - y^3\, dy$$

$$= 2\pi\left[\frac{2}{3}y^{3/2} - \frac{y^3}{3} + \frac{2}{5}y^{5/2} - \frac{y^4}{4}\right]\Bigg|_0^1 = 2\pi\left[\frac{2}{3} - \frac{1}{3} + \frac{2}{5} - \frac{1}{4}\right] =$$

$$= 2\pi\left(\frac{29}{60}\right) = \frac{29}{30}\pi.$$

$\boxed{Q\ \#3}$



At height x: $\therefore$ mass of Rope is

$(13-x)\cdot(0.9) = 11.7 - (0.9)x$

$\therefore$ The mass of water is: $g(x) = mx+n$

$m = \frac{0-38}{13-0} = \frac{-38}{13}$

0	38
x	
13	0

So $g(x) = -\frac{38}{13}x + n$. From $g(0) = 38 \longrightarrow n = 38$, hence:

$g(x) = -\frac{38}{13}x + 38$

$\therefore$ The mass of bucket is still 12 Kg.

HENCE: total mass: $m = 11.7 - x(0.9) - \frac{38}{13}x + 38 + 12$

$= -x\left(0.9 + \frac{38}{13}\right) + 61.7.$

Cut the interval $[0,13]$ into n-subintervals of equal length: $\Delta x = \frac{13-0}{n}$. Get a sample point x_i^* in $[x_{i-1}, x_i]$. W_i = work to lift from x_{i-1} to x_i is $\approx (9.8)\left[61.7 - x\left(0.9 + \frac{38}{13}\right)\right]\Delta x$

So $W = \sum_{i=1}^{n} W_i$. Passing to limit $(n \to \infty)$ one has $W = \int_0^{13} (9.8)\left[61.7 - x\left(0.9 + \frac{38}{13}\right)\right] dx = \int_0^{13} (9.8)\cdot(61.7)$

$- 9.8 \int_0^{13} \left(0.9 + \frac{38}{13}\right) x \, dx = 7860.58 -$

$9.8\,(323.05) = 7860.58 - 3165.89$

$= 4694.69\ J$

~~[18] G. D. Mostow, *Fully reducible subgroups of algebraic groups*, Amer. J. Math. **78** (1956), 200–221.~~

~~[19] G. D. Mostow, *Some new decomposition theorems for semi-simple groups*, Amer. Math. Soc. **14** (1955), 31-51.~~

~~[20] G. D. Mostow, *On covariant covering of Klein spaces*, Amer. J. Math. **77** (1955), 247–278.~~

~~[21] J.-P. Rosay, *Automorphisms of* $\mathbb{C}^n$*, a survey of Andersén-Lempert theory and applications*, Contemp. Math., **222**, AMS, Providence, 1999.~~

~~[22] D. Varolin, *The density property for complex manifolds and geometric structures*, J. Geom. Anal. **11** (2001), no. 1, 135–160.~~

~~[23] D. Varolin, *The density property for complex manifolds and geometric structures II*, Internat. J. Math. **11** (2000), no. 6, 837–847.~~

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~~*E-mail address*: `Frank.Kutzschebauch@math.unibe.ch`~~

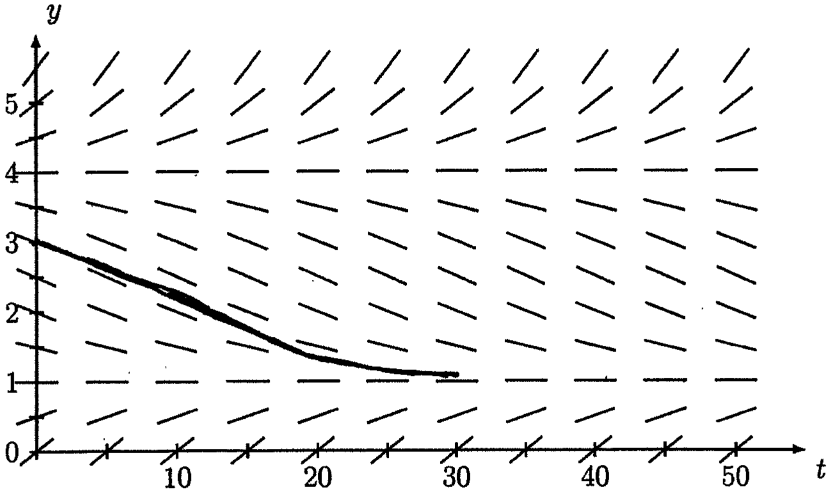
Questions 4 and 5 are short answer questions, for which you do not need to justify your answers.

4. [4 points] Consider the curve $x = t^2$, $y = e^{5t}$, $1 \leq t \leq 3$.
Write down the integral which gives the length of this curve. Simplify the integrand, but *do not calculate the integral.*

Response: $L = \int_1^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_1^3 \sqrt{(2t)^2 + (5e^{5t})^2}\ d$

$= \int_1^3 \sqrt{4t^2 + 25e^{10t}}\ dt$

5. [4 points] Below is the slope field for a differential equation $\frac{dy}{dt} = F(t, y)$.



(a) What are its equilibrium solutions (those of the form $y = C$)?

Response: $y = 4$, $y = 1$

(b) Draw, as well as you can, on the slope field, the graph of the particular solution with initial value $y(0) = 3$. Circle the answer below which is closest to $y(20)$.

Choices: (A) 0.8 [✓ (B) 1.3] (C) 2.2 (D) 3 (E) 4.5

6. [8 pts] Paul makes a cup of hot chocolate at the boiling temperature of water (100 °C), then puts it outside to cool it down. Outside, the temperature is $-15\,°C$, and 3 minutes later, when he checks his cup, its temperature is $41\,°C$.

(a) Let $T(t)$ be the temperature of the cup t minutes after Paul puts it outside. Supposing this temperature obeys Newton's law of cooling, give the differential equation which T satisfies, and solve it.

$$\frac{dT}{dt} = k\cdot(T-(-15)) = k(T+15)$$

$$\frac{dT}{T+15} = k\,dt \to \int \frac{1}{T+15}\,dT = \int k\,dt \Rightarrow \ln|T+15| = kt + C \text{ or}$$

$$|T+15| = e^{kt}\cdot e^{C} \text{ or } T+15 = Ae^{kt};\quad T = -15 + Ae^{kt}$$

FROM: $100 = T(0) \Rightarrow 100 = -15 + A$. So $\boxed{A = 115}$.

From: $41 = T(3) \Rightarrow 41 = -15 + 115\,e^{3k} \Rightarrow \frac{56}{115} = e^{3k}$ or:

$3k = \ln\left(\frac{56}{115}\right)$ or $k = \frac{1}{3}\ln\left(\frac{56}{115}\right)$.

We got: $T(t) = -15 + 115\cdot e^{t\cdot\frac{1}{3}\cdot\ln\left(\frac{56}{115}\right)}$

(b) If Paul leaves his cup outside, at what time t will his hot chocolate freeze? (Suppose that hot chocolate freezes at a temperature of $0\,°C$.)

Find t such that $T(t) = 0$; $0 = -15 + 115\cdot e^{t\cdot\frac{1}{3}\cdot\ln\left(\frac{56}{115}\right)}$

or $\frac{15}{115} = e^{t\cdot\frac{1}{3}\cdot\ln\left(\frac{56}{115}\right)}$ or $\ln\left(\frac{15}{115}\right) = t\cdot\frac{1}{3}\ln\left(\frac{56}{115}\right)$

So: $t = \dfrac{\ln\left(\frac{15}{115}\right)}{\frac{1}{3}\ln\left(\frac{56}{115}\right)} \approx \underline{8.43 \text{ min}}$

7. [8 pts] Solve the initial value problem $\frac{dy}{dt} = \frac{6t\sin(3t)}{y}$, $y(0) = 2$.
Express the solution y as a function of t.

$$y\,dy = 6t\sin(3t)\,dt \rightarrow \int y\,dy = \int 6t\sin(3t)\,dt \rightarrow \frac{y^2}{2} =$$

$$\left(\begin{array}{l} f'(t) = \sin 3t \rightarrow f(t) = \frac{-\cos(3t)}{3} \\ g(t) = 6t \rightarrow g'(t) = 6 \end{array} \right.$$

$$= -\frac{\cos(3t)}{3} \cdot 6t - \int -\frac{\cos(3t)}{3} \cdot 6\,dt$$

$$= \frac{-\cos(3t)}{3} \cdot 6t + 2\int \cos(3t)\,dt = -2t\cdot\cos(3t) + 2\cdot\frac{\sin 3t}{3} + C$$

C‡

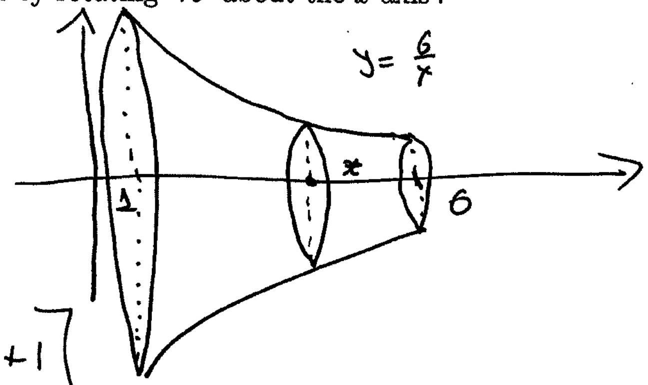
From $2 = y(0) \Rightarrow \frac{4}{2} = -0 + 0 + C \Rightarrow C = 2$

<u>So</u>: $y = +\sqrt{-4t\cos(3t) + \frac{4\sin(3t)}{3} + 2}$, since $\underline{y(0) > 0}$

8. [6 pts **bonus**] Let $\mathcal{R}$ be the region of the plane defined by $1 \le x \le 6$ and $0 \le y \le \frac{6}{x}$.
What is the volume of the solid of revolution obtained by rotating $\mathcal{R}$ about the x-axis?

Let $1 \le x \le 6$.

$$A(x) = \pi r^2 = \pi \left(\frac{6}{x}\right)^2 = \frac{\pi \cdot 36}{x^2}$$



$$V = \int_1^6 \frac{\pi\cdot 36}{x^2}\,dx = \pi\cdot 36\int_1^6 x^{-2}\,dx$$

$$= \pi\cdot 36 \cdot \left(\frac{x^{-1}}{-1} \Big|_1^6 \right) = \pi\cdot 36\left[-\frac{1}{6} + 1\right]$$

$$= \pi\cdot 36\cdot\frac{5}{6} = \boxed{30\pi}$$

1. (a) [4 pts] Why is $\int_0^6 \frac{x}{-x^2+9}\,dx$ an improper integral?

If it is convergent, compute its value. If not, explain why it is divergent.

It is improper because $\frac{x}{-x^2+9}$ is NOT defined AT $x=3$ in $[0,6]$

Note: $\int_0^3 \frac{x\,dx}{-x^2+9} = \lim_{t\to 3^-} \int_0^t \frac{x\,dx}{-x^2+9}\,dx = \lim_{t\to 3^-} \int_9^{-t^2+9} \frac{du}{(u)(-2)} =$

$= \lim_{t\to 3^-} \frac{1}{-2}\left[\ln|u| \Big|_9^{-t^2+9}\right] =$

$= \lim_{t\to 3^-} -\frac{1}{2}\left\{\ln|-t^2+9| + \ln 9\right\} = \infty$

$u = -x^2+9$; $\frac{du}{dx} = -2x$

$\frac{du}{-2} = x\,dx$

$x=0 \to u=9$

$x=t \to u=-t^2+9$

Conclusion: Our Integral is Divergent

(b) [4 pts] Determine if $\int_1^\infty \frac{3}{\sqrt{x}+7x^2}\,dx$ is convergent or divergent using an appropriate comparison test.

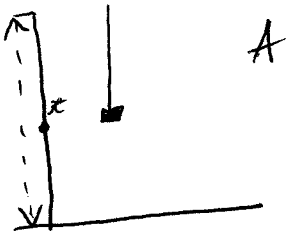
$\frac{3}{\sqrt{x}+7x^2} \le \frac{3}{0+7x^2} = \frac{3}{7}\cdot\frac{1}{x^2} \le \frac{1}{x^2}$ for all $x \ge 1$.

Since $\int_1^\infty \frac{1}{x^2}\,dx$ is Convergent $(p=2>1)$

one has by the Comparison Test that

our Integral is Convergent.

Q #3



At height x:

* mass of Rope is: $(13-x)\cdot(0.9) =$

$= 11.7 - x\cdot(0.9)$

* The mass of water is $g(x) = mx+n$ where $m = \frac{0-38}{13-0} = -38/13$.

0	38
x	?
13	0

So $g(x) = -\frac{38}{13}x + m$. Since $g(0) = 38 \Rightarrow n = 38$, or:

$g(x) = -\frac{38}{13}x + 38$.

* The mass of the bucket is still 11 Kg

Total mass is: $m = 11.7 - x(0.9) - \frac{38}{13}x + 38 + 11$

$= 60.7 - x\left(0.9 + \frac{38}{13}\right)$.

Cut the interval $[0, 13]$ into n subintervals of equal length: $\Delta x = \frac{13-0}{n}$. Get a sample point x_i^* in $[x_{i-1}, x_i]$. Let W_i = work to lift from x_{i-1} to x_i; $W_i \approx (9.8)\left[60.7 - x\left(0.9 + \frac{38}{13}\right)\right] \cdot \Delta x$. So $W = \sum_{i=1}^{n} W_i$, and by passing to limit $(n \to \infty)$ one has $W = \int_0^{13} (9.8)\left[60.7 - x\left(0.9 + \frac{38}{13}\right)\right] dx$

$= \int_0^{13} (9.8)(60.7)\,dx - \int_0^{13} 9.8\left(0.9 + \frac{38}{13}\right) x\,dx =$

$= 7733.18 - 9.8\,(323.05) =$

$= 7733.18 - 3165.89 = 4567.29$ J

8.7. **Corollary.** *Let an affine algebraic manifold X without nonconstant regular invertible functions possess an algebraic volume form ω such that under De Rham homomorphism ω corresponds to the zero element of $H^n(X, \mathbb{C}) = \mathbb{C}$. Then X cannot be a homogeneous space of any nontrivial reductive group.*

8.8. **Remark.** It was shown in [6] that any variety $X_{m,1} = \{x^m v + yu = 1\} \subset \mathbb{C}^4_{x,y,u,v}$ with $m \geq 2$ is diffeomorphic (as a real manifold) but not isomorphic to $X_{1,1} \simeq SL_2$ [8] because the unique (up to a constant factor) volume form $\omega_m = x^{-m} \mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}u$ on $X_{m,1}$ is exact ($\omega_m = \mathrm{d}\tau$ where $\tau = \frac{\mathrm{d}y \wedge \mathrm{d}u}{(1-m)x^{m-1}}$) . Corollary 8.7 enables us to tell now more: $X_{m,1}$ is not isomorphic to a homogeneous space of a nontrivial reductive group.

REFERENCES

[1] E. Andersén, *Volume-preserving automorphisms of* $\mathbb{C}^n$, Complex Variables Theory Appl. **14** (1990), no. 1-4, 223–235.

[2] E. Andersén, L. Lempert, *On the group of holomorphic automorphisms of* $\mathbb{C}^n$, Invent. Math. **110** (1992), no. 2, 371–388.

[3] I. Arzhantsev, H. Flenner, S. Kaliman, F. Kutzschebauch, M. Zaidenberg, *Flexible varieties and automorphism groups*, preprint, arXiv:1011.5375, 41p.

[4] F. Donzelli, *Algebraic Density Property of Homogeneous Spaces*, PhD thesis, University of Miami, (2009).

[5] F. Donzelli, A. Dvorsky, S. Kaliman, *Algebraic density property of homogeneous spaces*, Transformation Groups, **15:3** (2010) 551-576.

[6] A. Dubouloz, D. Finston, *On exotic affine 3-spheres*, preprint, arXiv:1106.2900, 13p.

[7] F. Forstnerič, J.-P. Rosay, *Approximation of biholomorphic mappings by automorphisms of* $\mathbb{C}^n$, Invent. Math. **112** (1993), no. 2, 323–349.

[8] T. Fujita, *On topology of non-complete algebraic surfaces*, J. Fac. Sci. Univ. Tokyo **29** (1982), 503-566.

[9] A. Grothendieck, *On the de Rham cohomology of algebraic varieties*, Inst. Hautes Etudes Sci. Publ. Math., **29** (1966), 95-105.

[10] P. Heinzner, *Geometric invariant theory on Stein spaces*, Math. Ann. **289** (1991), no. 4, 631-662.

[11] S. Kaliman, F. Kutzschebauch *Criteria for the density property of complex manifolds*, Invent. Math. **172** (2008), no. 1, 71–87.

[12] S. Kaliman, F. Kutzschebauch, *Density property for hypersurfaces* $uv = p(\bar{x})$, Math. Z. **258** (2008), no. 1, 115–131.

[13] S. Kaliman, F. Kutzschebauch, *On the present state of the Andersen-Lempert theory*, 42p., accepted to the Proceedings of the Russellfest.

[14] S. Kaliman, F. Kutzschebauch, *Algebraic volume density property of affine algebraic manifolds*, Invent. Math. **181:3** (2010) 605-647.

[15] Sh. Kobayashi, K. Nomizu, *Foundations of differential geometry* Vol. I. Reprint of the 1963 original. Wiley Classics Library. A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1996.

[16] F. Kutzschebauch, M. Leuenberger, *Lie algebra generated by locally nilpotent derivations on Danielewski surfaces*, preprint.

[17] Y. Matsushima, *Espaces homogènes de Stein des groupes de Lie complexes. I*, Nagoya Math. J. **16** (1960) 205-218.

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[8] Such varieties $X_{m,1}$ with $m \geq 2$ are examples of "complex exotic affine 3-spheres" since SL_2 is isomorphic to $\{x^2 + y^2 + u^2 + v^2 = 1\} \subset \mathbb{C}^4$.

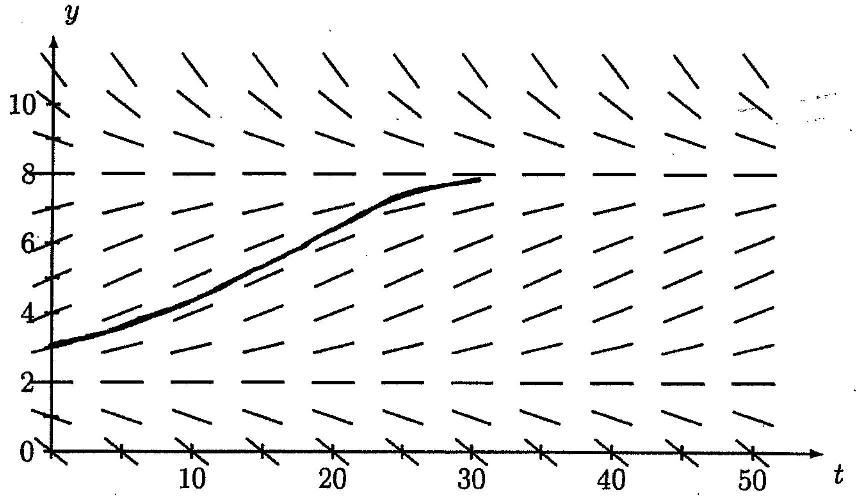
Questions 4 and 5 are short answer questions, for which you do not need to justify your answers.

4. [4 points] Consider the curve $x = t^2$, $y = e^{5t}$, $1 \le t \le 4$.
Write down the integral which gives the length of this curve. Simplify the integrand, but *do not calculate the integral.*

Response: $L = \int_1^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_1^4 \sqrt{(2t)^2 + (5e^{5t})^2}\, dt$

$= \int_1^4 \sqrt{4t^2 + 25e^{10t}}\, dt$

5. [4 points] Below is the slope field for a differential equation $\frac{dy}{dt} = F(t, y)$.



(a) What are its equilibrium solutions (those of the form $y = C$)?

Response: $y = 8$; $y = 2$

(b) Draw, as well as you can, on the slope field, the graph of the particular solution with initial value $y(0) = 3$. Circle the answer below which is closest to $y(20)$.

Choices: (A) 0.8 (B) 1.8 (C) 2.2 (D) 4 [✓ (E) 6.2]

6. [8 pts] Paul makes a cup of hot chocolate at the boiling temperature of water (100 °C), then puts it outside to cool it down. Outside, the temperature is −20 °C, and 3 minutes later, when he checks his cup, its temperature is 61 °C.

(a) Let $T(t)$ be the temperature of the cup t minutes after Paul puts it outside. Supposing this temperature obeys Newton's law of cooling, give the differential equation which T satisfies, and solve it.

$$\frac{dT}{dt} = k(T-(-20)) = k(T+20).$$

$$\frac{dT}{T+20} = k\,dt \Rightarrow \int \frac{1}{T+20} = \int k\,dt \Rightarrow \ln|T+20| = kt + C \Rightarrow$$

$$|T+20| = e^{kt}\cdot e^{C} \quad \text{or} \quad T+20 = Ae^{kt} \Rightarrow T = -20 + Ae^{kt}$$

From $100 = T(0) \Rightarrow 100 = -20 + A \Rightarrow \boxed{A = 120}$

From: $61 = T(3) \Rightarrow 61 = -20 + 120e^{3k} \Rightarrow \frac{81}{120} = e^{3k} \Rightarrow$

$$\ln\left(\frac{81}{120}\right) = 3k \Rightarrow k = \frac{1}{3}\ln\left(\frac{81}{120}\right)$$

We got: $T(t) = -20 + 120\,e^{t\cdot\frac{1}{3}\ln\left(\frac{81}{120}\right)}$

(b) If Paul leaves his cup outside, at what time t will his hot chocolate freeze? (Suppose that hot chocolate freezes at a temperature of 0 °C.)

find t such that: $0 = T(t)$; $0 = -20 + 120e^{t\cdot\frac{1}{3}\cdot\ln\left(\frac{81}{120}\right)}$

$$\Rightarrow \frac{20}{120} = e^{t\cdot\frac{1}{3}\cdot\ln\left(\frac{81}{120}\right)} \Rightarrow \ln\left(\frac{20}{120}\right) = t\cdot\frac{1}{3}\cdot\ln\left(\frac{81}{120}\right)$$

or: $$t = \frac{\ln\left(\frac{20}{120}\right)}{\frac{1}{3}\ln\left(\frac{81}{120}\right)} \approx \underline{13.68 \text{ min}}$$

7. [8 pts] Solve the inidial value problem $\frac{dy}{dt} = \frac{2t\cos(4t)}{y}$, $y(0) = 3$.
Express the solution y as a function of t.

$$y\,dy = 2t\cos(4t) \to \int y\,dy = \int 2t\cos 4t\,dt \to \frac{y^2}{2} = 2t \cdot \frac{\sin(4t)}{4}$$

$$f'(t) = \cos 4t$$
$$g(t) = 2t \to g'(t) = 2,\ f(t) = \frac{\sin(4t)}{4}$$

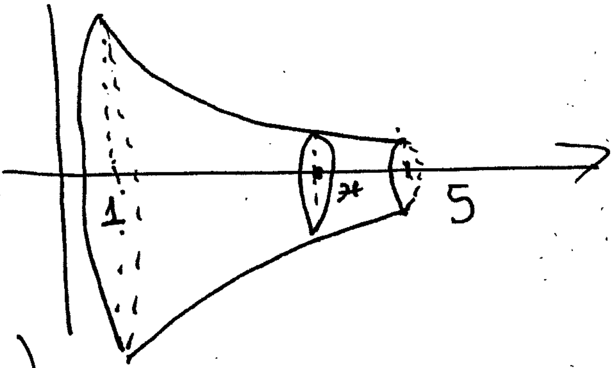
$$-\int 2 \cdot \frac{\sin(4t)}{4}\,dt = t \cdot \frac{\sin(4t)}{2} - \frac{1}{2}\int \sin(4t)\,dt = t\frac{\cdot \sin(4t)}{2} -$$

$$-\frac{1}{2}\left(-\frac{\cos(4t)}{4}\right) + C;\ C\#$$ From $3 = y(0) \Rightarrow \frac{9}{2} = \frac{1}{8} + C$

or: $C = \frac{9}{2} - \frac{1}{8} = \frac{35}{8}$.

$$y = \sqrt{t \cdot \sin(4t) + \frac{2 \cdot \cos(4t)}{8} + \frac{35}{8} \cdot 2}$$ since $y(0) = 3 > 0$

8. [6 pts **bonus**] Let $\mathcal{R}$ be the region of the plane defined by $1 \le x \le 5$ and $0 \le y \le \frac{5}{x}$.
What is the volume of the solid of revolution obtained by rotating $\mathcal{R}$ about the x-axis?



Let $1 \le x \le 5$.

$$A(x) = \pi r^2 = \pi\left(\frac{5}{x}\right)^2 = \frac{\pi \cdot 25}{x^2}$$

$$V = \int_1^5 \frac{\pi \cdot 25}{x^2}\,dx = \pi \cdot 25 \int_1^5 x^{-2}\,dx$$

$$= \pi \cdot 25 \cdot \left(\frac{x^{-1}}{-1}\Big|_1^5\right) = \pi \cdot 25\left(-\frac{1}{5} + 1\right)$$

$$= \pi \cdot 25 \cdot \frac{4}{5} = 20\pi$$