Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

Calculus III for Engineers
MAT 2322A - Fall 2011
Midterm II
Professor: Victor G. LeBlanc
Time limit: 80 minutes. Closed books. No calculators.

Name:_____ ID Number:_____

Instructions

- This exam has 9 pages and you have 80 minutes to complete it.
- This is a closed book exam. Furthermore, all calculators, cell phones, pagers or any other electronic or communication devices are forbidden.
- Read each question carefully before answering.
- All questions are long-answer questions. **A correct answer requires a full, clearly-written and detailed solution.** Answer each question in the space provided, using backs of pages or the extra pages at the end if necessary.
- Each question is worth 5 marks. There are 2 bonus marks associated with question 5.
- Do not unstaple the test.
- Good luck!

Grid below is used for grading
(do not write in this grid)

1	2	3	4	5	Total
/5	/5	/5	/5	/7	/25

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1. Consider the solid in the first octant bounded by the planes $z = 0$, $x = 0$, $y = 0$, $x = 1$, $y = 2$ and the paraboloid $z = 9 - x^2 - y^2$. This solid has a mass density given by the function $\delta(x, y, z) = xy$. Find the total mass of this solid.

2. Convert the following integral into polar coordinates. **DO NOT EVALUATE THE INTEGRAL**.

$$\int_0^{\frac{\sqrt{2}}{2}} \int_y^{\sqrt{1-y^2}} (x+y)\,dx\,dy$$

Hint: Sketch the region of integration in the x-y plane, and then express this region in terms of polar coordinates.

3. Consider the solid drawn on the following page which is bounded by the hemisphere $z = \sqrt{1 - x^2 - y^2}$, the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 0$. This solid has a mass density given by the function $\delta(x, y, z) = x^2 + y^2$. Using either spherical coordinates or cylindrical coordinates (your choice), set up a triple integral which gives the total mass of this solid. **DO NOT EVALUATE THE INTEGRAL**.

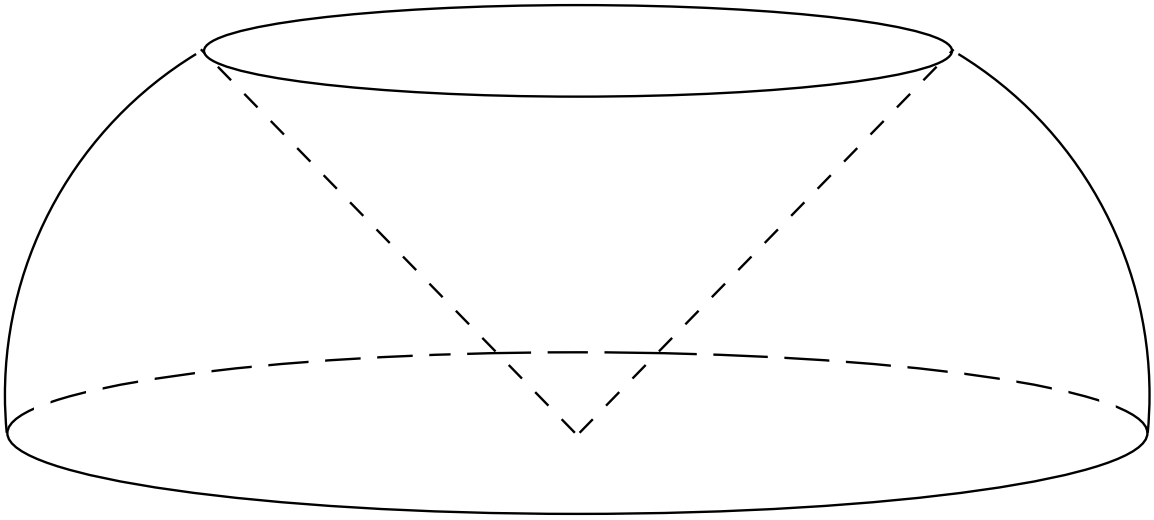

Figure 1: Figure representing the solid region described in problem 3.

4. Find the total arclength of the curve which is parametrized by the following vector function

$$\vec{r}(t) = \cos t\,\vec{\mathbf{i}} + t\,\vec{\mathbf{j}} + \sin t\,\vec{\mathbf{k}}, \quad 0 \leq t \leq 4\pi.$$

5. Let D be the unit disk in the x-y plane, i.e.

$$D = \{\, (x, y) \,|\, 0 \leq x^2 + y^2 \leq 1 \,\}$$

and let S be the graph of the function $z = 4 + y^2 - x^2$ defined for points (x, y) in D.

(a) Give a parametrization of this surface S.

(b) Set up an integral which would give the total area of this surface S, **but do not evaluate this integral**.

(c) **BONUS [2 marks]** Evaluate the integral in (b). Note that you are eligible to receive bonus marks **only if you have the correct answer in (b)**.

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