WHITE

Q1. $P = \$3\,000$

$r = 1.1\% = 0.011$

$F = P(1+r)^3 = 3000(1+0.011)^3 = \$3\,100$

Q2. $F = \$15\,000$

$r = 5\% = 0.05$

$P = \dfrac{F}{(1+r)^3} = \dfrac{15000}{(1+0.05)^3} = \$12\,958$

Q3. $i_e = 16.2\% = 0.162$

$m = 365$

$r = ?$

$\therefore \quad i_e = \left(1+\frac{r}{m}\right)^m - 1$

$\Rightarrow 0.162 = \left(1+\frac{r}{365}\right)^{365} - 1$

$\Rightarrow 1+0.162 = \left(1+\frac{r}{365}\right)^{365} - 1 + 1$

$\Rightarrow 1.162 = \left(1+\frac{r}{365}\right)^{365}$

$\Rightarrow (1.162)^{\frac{1}{365}} = \left(1+\frac{r}{365}\right)^{365 \times \frac{1}{365}}$

$\Rightarrow 1+\frac{r}{365} = \sqrt[365]{1.162}$

$\Rightarrow 1+\frac{r}{365} = 1.000411$

$\Rightarrow \frac{r}{365} = 0.000411 \quad \Rightarrow r = 365 \times 0.000411 \Rightarrow r = 15\%$

Q4. $r = 6\% = 0.06$

$m = 4$

- So, quarterly nominal interest rate $i_s = \frac{r}{m} = \frac{6\%}{4} = 1.5\%$
$= 0.015$
- For effective annual interest rate i_e, you need to observe that, in a year, interest rate compounds 4 times as it is compounded quarterly.
Hence, $m = 4$ for $i_e = (1+i_s)^m - 1$

$$\Rightarrow i_e = (1+0.015)^4 - 1$$
$$= 1.0613636 - 1$$
$$= 0.0613636$$

- Now total interest for $(N=)$ 5 years $I = \$2081.13$
and $P = ?$
- So, $(P+I) = P(1+i_e)^5$ where, $P+I = F$

$$\Rightarrow P + 2081.13 = P(1+0.0613636)^5$$
$$\Rightarrow 2081.13 = P\{(1.0613636)^5 - 1\}$$
$$\Rightarrow P = \frac{2081.13}{\{\cdot\}}$$
$$\therefore P = \$6000$$

Q5. Simple interest rate, $i = 20\% = 0.2$

$P = 2000$

$N = 5$

So the total amount of interest over 5 yrs is:

$$I_s = PiN = 2000 \times 0.2 \times 5$$
$$= 400 \times 5 = 2{,}000$$

Q6. • Principal amount to be borrowed, $P = 1000$

• Effective interest rate offered by Bank A, $i_{eA} = 5\% = 0.05$

• Nominal interest rate offered by Bank B, $i_{sB} = \frac{5\%}{12}$ compounded monthly

[Hence, for Bank B, $r_B = 5\% = 0.05$ and $m = 12$]

• Thus, the effective interest rate offered by Bank B is,

$$i_{eB} = \left(1 + \frac{r_B}{m}\right)^m - 1 \quad \text{compounded annually}$$
$$= \left(1 + \frac{0.05}{12}\right)^{12} - 1$$
$$= (1 + 0.004167)^{12} - 1$$
$$= (1.004167)^{12} - 1$$
$$= 1.051162 - 1$$
$$\Rightarrow i_{eB} = 0.051162$$

Now, if you borrow from Bank A, your total interest payment over N compounding period would be

$$P(1+i_{eA})^N - P = 1000(1+0.05)^N - 1000$$
$$= 1000\left[1.05^N - 1\right] \text{ ——— [eq A]}$$

If instead you borrow from Bank B, your total interest payment over N compounding period would be

$$P(1+i_{eB})^N - P - 100 = 1000(1+0.051162)^N - 1000 - 100$$
$$= 1000\left[1.051162^N - 1\right] - 100 \text{ ——— [eq B]}$$

where, the value of the free cell phone (i.e. the promotional offer by Bank B) is substracted

- Given [eq A] and [eq B], for you to choose Bank B it must be that your total interest payment over N compounding period to Bank B <u>must be lower</u> than what you would pay to Bank A.

[eq B] < [eq A]

Hence, $\Rightarrow 1000\left[1.051162^N - 1\right] - 100 < 1000\left(1.05^N - 1\right)$

$$\Rightarrow 1000\left[1.051162^N - 1\right] - 1000\left(1.05^N - 1\right) < 100$$

$$\Rightarrow 10\left[1.051162^N - 1\right] - 10\left(1.05^N - 1\right) < 1$$

[Diving both sides by 100]

$$\Rightarrow 10\left\{\left[1.051162^N - 1\right] - \left(1.05^N - 1\right)\right\} < 1$$

$$\Rightarrow \left\{1.051162^N - \cancel{1} - 1.05^N + \cancel{1}\right\} < \frac{1}{10}$$

[opened up the brackets on L.S. &' divided both sides by 10]

$$\Rightarrow \boxed{1.051162^N - 1.05^N < 0.1} \quad \text{—— [eq C]}$$

- As long as this condition holds for any value of N, Bank B is the better choice.

- Based on the options provided for answers, it turns out that the longest duration of N for which this condition in [eqⁿ C] holds is <u>N = 25 yrs</u>

- Check for N = 25, [eq C] is $0.094944 < 0.1 \Rightarrow$ eqⁿ holds
  but N = 30, [eq C] is $0.145815 \not< 0.1 \Rightarrow$ condradiction

Q7. Answer is D

Q8. The question specifies nominal interest of 18% compounding quarterly AND is asking what would be the equivalent nominal interest rate per quarter. Hence it is $\frac{18\%}{4}$ where $m=4$

$= 4.5\%$

Q9. • Note that Emily's principal amount $P = 4000$

• The Savings Account (SA) pays an effective interest rate of $i_{e,SA} = 4.6\% = 0.046$ (annually)

• The Investment Certificate (IC) pays a nominal monthly interest of $i_{s,IC} = 0.3\% = 0.003$

Hence, for IC the effective annual interest rate is,

$$i_{e,IC} = (1 + i_{s,IC})^m - 1, \text{ where } m = 12$$
$$= (1 + 0.003)^{12} - 1$$
$$= 1.003^{12} - 1$$
$$= 1.0366 - 1 = 0.0366 \text{ (annually)}$$

Now we are ready to calculate the total interest payment that could be earned from each option.

If SA is chosen: $P(1 + 0.046)^1 - P$

$$= 4000\left[1.046^1 - 1\right]$$
$$= 4000 \times 0.046 = \$184$$

$\boxed{4}$ If IC is chosen: $P(1+0.0366)^1 - P$

$= 4000[1.0366 - 1]$

$= 4000 \times 0.0366$

$= \$146.4$

Clearly then, by putting money into SA, Emily gains $(184 - 146.4) = \$37.6$ by the end of the year.

Q10. Answer is C. Why?

- Note that all projects are <u>independent</u> (not mutually exclusive).
- All projects have IRR (not incremental IRR) at least equal to or greater than MARR of 16% except for project 2. So, project 2 is never considered.
- Since all projects are independent, all projects with IRR $\geq$ MARR should be undertaken if the company is not contraint by cash flow for First cost. Note that the company has \$1,025,000 and the total first cost for project 1, 3, 4 and 5 together is \$850,000. Hence, the company should undertake <u>Projects 1, 3, 4 and 5.</u>

Q12. This question required you to make use of, Series Present Worth Factor $(P|A,i,N) = \left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$

where, $i = MARR = 15\% = 0.15$

$N = 15$ years

so, $(P|A,i,N) = \left[\frac{(1+0.15)^{15} - 1}{0.15(1+0.15)^{15}}\right]$

$= \left[\frac{1.15^{15} - 1}{0.15 \times 1.15^{15}}\right]$

$= 5.84737$

- Senex pension plan requires an investment of \$1500 a year for 15 years. Thus, $A_S = 1500$, where A_S is the annuity amount for Senex pension plan.

  Hence, the present cost of Senex pension plan is,

  $P_S = A_S \cdot (P|A,i,N)$

  $= 1500 \times 5.84737$

  $= \$8,771.06$

- Geriatrix pension plan requires an immediate deposit of \$5000 and subsequent annual investment of \$1200. Hence, $A_G = 1200$

  Therefore, the present cost of Geriatrix pension plan is,

  $P_G = 5000 + A_G \cdot (P|A,i,N)$

  $= 5000 + 1200 \times 5.84737$

  $= \$12,016.84$

- Thus, the Geriatrix pension plan costs an additional present cost of $(12016.84 - 8771.06) \approx \$3,246$.

Q13. Answer is B.

Q14. This question required the application of the concept of <u>repeated lives</u> since:

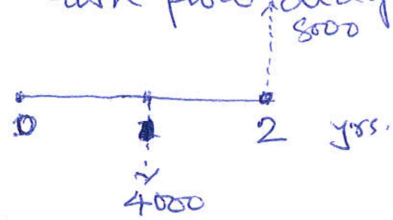
i) the working lives of three different lawnmower is different (i.e., 3, 4, 5 yrs.); and

ii) the technology for foreseeable future is assumed to be stable.

Hence, you needed to find the <u>least common multiplier</u> for 3, 4, 5 yrs; which is <u>60 years</u>.

Q15. Answer is **E**.

Q16. For the question, the cash flow diagram looks like the following:



So, the sum of the discounted cash out flow (or, disbursement) is,

$$\sum_{t=0}^{2} \frac{D_t}{(1+i)^t} = D_0 + \frac{D_1}{(1+i)} + \frac{D_2}{(1+i)^2} = 0 + \frac{4000}{(1+i)} + \frac{0}{(1+i)^2} = \frac{4000}{(1+i)}$$

And, the sum of the discounted cash in-flow (or, receipts) is,

$$\sum_{t=0}^{2} \frac{R_t}{(1+i)^t} = R_0 + \frac{R_1}{(1+i)} + \frac{R_2}{(1+i)^2} = 0 + 0 + \frac{8000}{(1+i)^2}$$

where, $i = IRR$.

By definition of IRR:

$$\frac{4000}{(1+i)} = \frac{8000}{(1+i)^2}$$

$$\Rightarrow \frac{1}{(1+i)} = \frac{2}{(1+i)^2}$$

$$\Rightarrow (1+i)^2 = 2(1+i)$$

$$\Rightarrow 1+i = 2 \Rightarrow i = 1 = 100\%$$

Q18. • Initial investment is \$12,000
- Annual maintenance cost is \$1,000
- Annual revenue is \$2,600
- Project life is 10 yrs.

Note that net annual return/revenue is \$(2600-1000) = \$1,600

By definition, payback period is the amount of time required for a project to get its initial investment/cost recouped.

Hence, the payback period is $\frac{\$12,000}{\$1,600} = 7.5$ yrs.

Q19. Answer is E.

Q20. ~~Compounded~~
- Note that question states 12% annual interest rate compounded monthly.
- Hence the effective monthly interest rate is $\frac{12\%}{12} = 1\%$.
- Future sum, $F = \$10,000$
- No. of compounding period = 3 yrs = 36 months
- To find out how much one needs to set aside each month to accumulate \$10,000 over 36 months, we need to make use of the Sinking Fund Factor,

$$(A|F,i,N) = \frac{i}{(1+i)^N - 1}$$

where, $F = \$10,000$
$i = 1\% = 0.01$
$N = 36$
$A = ?$

Recall, that $(A|F, i, N)$ converts $F \rightarrow A$.

Hence we need to solve:

$$A = F \cdot (A|F, i, N)$$

$$= 10{,}000 \times \frac{0.01}{\{(1+0.01)^{36} - 1\}}$$

$$= 10{,}000 \times \frac{0.01}{\{(1.01)^{36} - 1\}}$$

$$\therefore \quad A = \$232.14$$

Q21. Answer is E.

Q22. • Jen. lends, $P = \$2{,}000$

• Interest rate, $i = 10\% = 0.1$ compounding annually
• Compounding period / Loan period = 4 yrs.
• Total interest to be had, $I = ?$

• Future worth, $F = ?$

Thus, $F = P(1+i)^N = 2000(1+0.1)^4$

$$= 2000 \times (1.1)^4$$

$$= 2000 \times 1.4641$$

$$= 2928.2$$

Hence, $I = F - P = (2{,}928.2 - 2{,}000)$

$$= 928.2 \approx \$928$$

Q23. • Future worth, $F = \$1,000,000$

- Annuity (i.e., yearly deposit), $A = \$5,000$
- Interest rate, $i = 5\% = 0.05$
- How much time required (i.e., compounding period), $N = ?$

Note that the question asks you to convert an annuity (A) into future worth (F) over the compounding period (N) $\Rightarrow A \to F$

Hence you need to use, the uniform series compound amount factor:

$$(F|A, i, N) = \frac{(1+i)^N - 1}{i}$$

Thus, you need to solve:

$$F = A \cdot (F|A, i, N)$$

$$\Rightarrow 1000\,000 = 5\,000 \times \frac{\{(1+0.05)^N - 1\}}{0.05}$$

$$\Rightarrow \frac{1000\,\cancel{000}}{5\,\cancel{000}} = \frac{(1.05)^N - 1}{0.05}$$

$$\Rightarrow 1000 \times 0.05 = (1.05)^N - 1$$

$$\Rightarrow 10 = (1.05)^N - 1$$

$$\Rightarrow (1.05)^N = 11$$

$$\Rightarrow N \ln(1.05) = \ln 11$$

$$\therefore N = \frac{\ln 11}{\ln 1.05} \approx 50 \text{ yrs.}$$

Q24. • Grapefruit comp. costs \$3,000

• Doors comp. costs \$2,500

The difference in cost is \$(3000 − 2500) = \$500

• Salvage value of Grapefruit comp after 4 yrs is \$200

Hence, the present worth of that salvage value at MARR (i) = 10% = 0·1 is:

$$P = \frac{200}{(1+i)^n} = \frac{200}{(1+0{\cdot}1)^4} = \frac{200}{(1{\cdot}1)^4} = 136{\cdot}6$$

Thus, the present cost of choosing Grapefruit comp is: \$(500 − 136·6) = \$363·4 ≈ \$363

Q25. Answer is D.
