MAT 2379, Introduction to biostatistics

Solution to Assignment 1
Due date: Monday September 24, 2012

$Total = 100$ marks

Problem 2.4 (15 marks) (a) Both parents have the genotype Ff.

Female Gamete	Male Gamete $\frac{1}{2}F$	Male Gamete $\frac{1}{2}f$
$\frac{1}{2}F$	$\frac{1}{4}FF$ (frizzled)	$\frac{1}{4}Ff$ (slightly frizzled)
$\frac{1}{2}f$	$\frac{1}{4}Ff$ (slightly frizzled)	$\frac{1}{4}ff$ (normal)

The offspring can be frizzled with probability 1/4, normal with probability 1/4, and slightly frizzled with probability 1/2.
(b) Male = Ff; Female = ff.

Female Gamete	Male Gamete $\frac{1}{2}F$	Male Gamete $\frac{1}{2}f$
ff	$\frac{1}{2}Ff$ (slightly frizzled)	$\frac{1}{2}ff$ (normal)

The offspring can be slightly frizzled with probability 1/2, and normal with probability 1/2.

(b) Male = Ff; Female = FF.

Female Gamete	Male Gamete $\frac{1}{2}F$	Male Gamete $\frac{1}{2}f$
FF	$\frac{1}{2}FF$ (frizzled)	$\frac{1}{2}Ff$ (slightly frizzled)

The offspring can be slightly frizzled with probability 1/2, and frizzled with probability 1/2.

Problem 3.1 (15 marks) Let A be the event that the chicken has fatty liver syndrome, and B the event that the chicken has cage layer fatigue. We know that $P(A) = 0.02$, $P(B) = 0.03$ and $P((A' \cap B) \cup (A \cap B')) = 0.025$.
(a)

$$0.05 = P(A) + P(B) = P((A' \cap B) \cup (A \cap B')) + 2P(A \cap B) = 0.04 + 2P(A \cap B)$$

Hence

$$P(A \cap B) = \frac{0.05 - 0.04}{2} = 0.005$$

(b)

$$P(A' \cap B') = 1 - P(A \cup B) =$$

$$1 - [P(A) + P(B) - P(A \cap B)] = 1 - [0.02 + 0.03 - 0.005] = 1 - 0.045 = 0.955$$

Problem 3.4 (10 marks) Let A be the event that the subject has the disease and B the event that the subject is exposed to the risk factor. We know that $P(A) = 0.23$, $P(B) = 0.19$, $P(A \cup B) = 0.31$.
(a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.23 + 0.19 - 0.31 = 0.11$.
(b) $P(A' \cap B) = P(B) - P(A \cap B) = 0.19 - 0.11 = 0.08$.
(c) $P(A \cap B') = P(A) - P(A \cap B) = 0.23 - 0.11 = 0.12$.

Problem 4.2 (10 marks) (a) The false positive rate is: $P(\text{test}+|\text{true}-) = 50/995 = 0.05$. The false negative rate is: $P(\text{test}-|\text{true}+) = 2/5 = 0.4$.

(b) The sensitivity is $P(\text{test}+|\text{true}+) = 3/5 = 0.6$. The specificity is $P(\text{test}-|\text{true}-) = 945/995 = 0.95$

(c) The positive predictive value is $P(\text{true}+|\text{test}+) = 3/53 = 0.06$. The positive predictive value is $P(\text{true}-|\text{test}-) = 945/947 = 0.998$.

Problem 5.2 (10 marks) The probability of survival is $(266 - 57)/266 = 209/266 = 0.7857$. The probability that all 5 birds will survive is $(0.7857)^5 = 0.299$

The probability of dying is $57/266 = 0.2143$. The probability that all 5 will die is $(0.2143)^5 = 0.00045$.

Problem 5.6.(a) (15 marks) Let C be the event of having cancer. We know that $P(C) = 0.05$ We also know that the false positive rate is $P(T+|U-) = 0.05$ and that the false negative rate is $P(T-|U+) = 0.04$. Thus, $P(T-|C') = 0.95$ and $P(T+|C) = 0.96$.
(a) By Bayes' rule,

$$
\begin{aligned}
P(C|T+) = \frac{P(C \cap T+)}{P(T+)} &= \frac{P(T+|C)P(C)}{P(T+|C)P(C) + P(T+|C')P(C')} \\
&= \frac{(0.96)(0.05)}{(0.96)(0.05) + (0.05)(0.95)} \\
&= 0.5026.
\end{aligned}
$$

Problem 8.2 (10 marks) (a) There are 4 possible cases: (1) $I^A I^A \times I^B I^B$; (2) $I^A I^A \times I^B i$; (3) $I^A i \times I^B I^B$; (4) $I^A i \times I^B i$.
(b) Case (1): $I^A I^A \times I^B I^B$

Female Gametes	Male Gametes I^B
I^A	$I^A I^B$ (type AB)

Case (2): $I^A I^A \times I^B i$

Female Gametes	Male Gametes $\frac{1}{2} I^B$	$\frac{1}{2} i$
I^A	$\frac{1}{2} I^A I^B$ (type AB)	$\frac{1}{2} I^A i$ (type A)

Case (3): $I^A i \times I^B I^B$

Female Gametes	Male Gametes I^B
$\frac{1}{2} I^A$	$\frac{1}{2} I^A I^B$ (type AB)
$\frac{1}{2} i$	$\frac{1}{2} i I^B$ (type B)

Case (4): $I^A i \times I^B i$

Female Gametes	Male Gametes $\frac{1}{2}I^B$	$\frac{1}{2}i$
$\frac{1}{2}I^A$	$\frac{1}{4}I^A I^B$ (type AB)	$\frac{1}{4}I^A i$ (type A)
$\frac{1}{2}i$	$\frac{1}{4}iI^B$ (type B)	$\frac{1}{4}ii$ (type O)

(c) Yes, it is possible that their offspring has type O blood. This happens in Case (4).

Problem 8.16 (15 marks) (a) $P(GGG) = P(G)P(G)P(G) = (0.5)^3 = 0.125$.

(b) Let A be the event that all three are girls and B be the event that the oldest and the youngest are girls. Note that $A \cap B = A$, thus

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{0.125}{0.25} = 0.5,$$

since

$$P(B) = P(GGG) + P(PGB) = (0.5)^3 + 0.5^3 = 0.25.$$

(c) Let C be the event of all boys. The probability that at least one child is a girl is

$$P(C') = 1 - P(C) = 1 - P(\text{boy})P(\text{boy})P(\text{boy}) = 1 - (0.5)^3 = 0.875.$$