

MATH 209/4 all sections except EC: - Fundamental Mathematics II  
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Midterm - Sunday, March 2, 2014 (1h30min) 14:00-15:30  
Only approved calculators are permitted.

MARKS

- [7] 1. (a) Find  $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x^2 - x - 6}$ .
- [8] (b) Give an example of a function  $g(x)$  and a function  $h(x)$  with the following properties:
- (i)  $\lim_{x \rightarrow 7} g(x) = 0$                       (ii)  $\lim_{x \rightarrow 7} h(x) = 0$                       (iii)  $\lim_{x \rightarrow 7} \frac{[g(x)]^2}{h(x)} = 2$
- [7] 2. Let  $k(x) = x^3 - 5$ . Work out the following in detail:  $\lim_{s \rightarrow 0} \frac{k(x+s) - k(x)}{s}$
- [12] 3. (a) If  $f(x) = 4x^{\frac{3}{4}} - x^{-6}$ , find  $f'(1)$ . You don't have to simplify the answer.
- (b) If  $g(x) = [4x^3 + 7] [3 - \ln(x^2)]$ , find  $g'(2)$ . You don't have to simplify the answer.
- (c) Find  $h'(x)$  if  $h(x) = \frac{x^2 - \frac{1}{x}}{e^x - x^2}$ . You don't have to simplify the answer.
- (d) Find the value of  $dy$  if  $y = \ln(x+1)$ ,  $x = 3$ , and the change in  $x$  is 0.2.
- [6] 4. A stock grew from \$35 to \$120,000 in 43 years. Assuming continuous compounding, what is the associated annual rate of growth?
- [10] 5. The total profit (in dollars) from the sale of  $x$  lawn mowers is  $P(x) = 30x - 0.03x^2 - 750$ ,  $0 \leq x \leq 1,000$ .
- (a) Find the average profit per mower if 50 mowers are produced.
- (b) Find the marginal average profit at a production level of 50 mowers, and interpret the results.
- (c) Use the results from parts (a) and (b) to estimate the average profit per mower if 51 mowers are produced.
- [10] 6. Find  $x'$  for the function  $x = x(t)$  defined implicitly by  $1 + x \ln t = te^x$  and evaluate  $x'$  at  $(t, x) = (1, 0)$ .
- [10] 7. A person who is new on an assembly line performs an operation in  $T$  minutes after  $x$  performances of the operation, as given by

$$T = 6 \left( 1 + \frac{1}{\sqrt{x}} \right)$$

If  $dx/dt = 6$  operations per hour, where  $t$  is time in hours, find  $dT/dt$  after 36 performances of the operation.