

COMP 232 Mathematics for Computer Science
Fall 2014
Midterm Exam II

Name: _____

Total Points:

ID: _____

_____ / 30

Instructions. *This is a closed book exam. The only allowed tool is an ENCS approved calculator. Provide all answers in this booklet. Use pen, not pencil. Do not detach any pages from this exam!*

(2^{pts}_{ea.}) 1. Let $f : \mathbb{Z} \rightarrow \mathbb{Z}$, and $g : \mathbb{Z} \rightarrow \mathbb{Z}$ be functions. What is the truth value of each of the following?

10 pts

(a) If f and g are both one-to-one, then the sum function $f + g$ is also one-to-one.

False

True

Don't know!

(b) If f and g are both onto, then the product function $f \cdot g$ is also one-to-one.

False

True

Don't know!

(c) If f and the function composition $f \circ g$ are onto, then g is onto.

False

True

Don't know!

(d) If f is onto and g is one-to-one, then the function composition $f \circ g$ is also onto.

False

True

Don't know!

(e) If f is onto and g is one-to-one, then the function composition $g \circ f$ is also onto.

False

True

Don't know!

10 pts

(2pts_{ea.}) 2. Which of the following statements are true, and which are false?

8 pts

(a) If $a \equiv b \pmod{m}$, and if $c \equiv d \pmod{m}$, where a, b, c, d and m are integers, with $m \geq 2$, then $a - c \equiv b - d \pmod{m}$.

True

False

Don't know!

(b) If $ac \equiv bc \pmod{m}$, where a, b, c , and m are integers, with $m \geq 2$, then $a \equiv b \pmod{m}$.

False

True

Don't know!

(c) If $n \mid m$ and $a \equiv b \pmod{m}$, then $a \equiv b \pmod{n}$, where n and m are positive integers greater than 1.

True

False

Don't know!

(d) If $a \equiv b \pmod{m}$, and if $c \equiv d \pmod{m}$, where a, b, c, d and m are integers, with $c > 0, d > 0, m \geq 2$, then $a^c \equiv b^d \pmod{m}$.

True

False

Don't know!

8 pts

- (2pts ea.) 3. (a) Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \lfloor \frac{x-1}{2} \rfloor$. Let $S = \{x \in \mathbb{R} : 1 \leq x \leq 6\}$, and $T = \{2\}$. In the box below, write the values of $f(S)$ and $f^{-1}(T)$

8 pts

$$f(S) = \dots\dots \text{Answer: } \{0, 1, 2\}$$

$$f^{-1}(T) = \dots\dots \text{Answer: } \{x \in \mathbb{R} : 5 \leq x < 7\}$$

- (b) Let A , and B be sets, with $|A| = 3$ and $|B| = 2$. How many of all the possible function $f : A \rightarrow B$ are onto, and how many are injections? Write your answer in the box below.

The number of onto functions is $\dots\dots$ Answer: 6

The number of injective functions is $\dots\dots$ Answer: 0

- (c) In the box below, prove that if $n \in \mathbb{Z}^+$ is odd, then $n^2 \equiv 1 \pmod{8}$.

Proof:

Since n is odd, $n = 2k + 1$, for some $k \in \mathbb{Z}$. Thus $n^2 = 4k^2 + 4k + 1 = 4k(k + 1) + 1$. Since either k or $k + 1$ is even, it follows that $8 \mid 4k(k + 1)$. Hence $n^2 \pmod{8} = 1$, and thus $n^2 \equiv 1 \pmod{8}$.

- (d) Let $x \in \mathbb{R}$, and $n \in \mathbb{Z}$. Fill in the blanks in statements 1–4 below

1. If $x \notin \mathbb{Z}$, then $\lceil x \rceil - \lfloor x \rfloor = \dots\dots$ Answer: 1

2. If $x \in \mathbb{Z}$, then $\lceil x \rceil - \lfloor x \rfloor = \dots\dots$ Answer: 0

3. $\lceil x + n \rceil = n + \dots\dots$ Answer: $\lceil x \rceil$

4. $\lfloor \frac{n}{2} \rfloor = \frac{n-1}{2}$ if n is $\dots\dots$ Answer: odd

8 pts

- (4pts) 4. In the box below, construct a proof of the following claim.
Hint: Use a proof by contradiction.

4 pts

*If a prime number is the sum of two prime numbers,
then one of the numbers equals 2.*

Proof:

Let p_1 and p_2 be prime numbers, with $p_1 + p_2 = p$.
Suppose neither p_1 nor p_2 is equal to 2.
Since all prime numbers except 2 are odd, it follows that both p_1 and p_2
must be odd (and greater than 2).
Hence $p = p_1 + p_2$ is even, and greater than 2.
This contradicts that p is prime.

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4 pts