ECO3150
Introduction to Probability and Statistics
University of Ottawa

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First midterm examination

There are two parts of this examination: one consisting of problems, and one consisting of written questions. Explain your work and show your calculations. You should use non-programmable calculators. It is strictly forbidden to invoke programmed statistical functions on a calculator or any other wireless device. Do not provide irrelevant information. Good luck.

A. Problems

1. A sample of 20 financial analysts was asked to produce forecasts of the price of a stock of a corporation for the next year. The results are tabulated in the following table.

Stock price forecast in $	9.95-10.45	10.45-10.95	10.95-11.45	11.45-11.95	11.95-12.45
Number of analysts	2	8	6	3	1

a) Give a sketch of the histogram.
b) Find the relative frequencies.
c) Find the cumulative frequencies.
d) Intrepret the cumulative frequency of the 4th cell ($11.45 - 11.95).

2. A chain store drew a random sample comprised of 10 of its own stores in the province. The question that they posed in their survey was the growth rate in % terms of total sales relative to the same period during the preceding year. These data appear in the table below.

10.2	3.1	5.9	7.0	3.7
2.9	6.8	7.3	8.2	4.3

a) Calculate the average value
b) Calculate the median value.
c) Comment on whether the distribution appears to be symmetric, right-skewed, or left-skewed, and explain your response.

3. Consider the following data.

6	8	7	10	3	5	9	8

a) Calculate the variance.
b) Calculate the standard error.
c) Calculate the coefficient of variation. In your judgement, is there a high degree of dispersion? Explain why or why not.

4. Consider the following sample of 24 observations for the random variable X_i. The W_i terms refer to the raw frequencies for each of the 5 values.

X_i	W_i
4.6	8
3.2	3
5.4	6
2.6	2
5.2	5

a) Calculate the simple arithmetic mean.
b) Calculate the weighted mean.
c) Explain the nature of the difference between them.

5. The following data pertain to a random variable denoted X_i (the price of an item) and the corresponding quantity demanded (Y_i).

Price	Quantity demanded
5	55
6	53
7	45
8	40
9	20

a) Calculate and interpret the covariance.
b) Calculate and interpret the coefficient de correlation

6. Consider the following 4 possible realisations (or outcomes O_i) for the TSX stock index (the Toronto Stock Exchange) over two consecutive days.

O_1: the index rises on both days
O_2: the index rises the first day but falls on the second day
O_3: the index falls the first day but rises on the second day
O_4: the index falls on both days

Also consider two events A et B.

A: The index rises on the first day
B: The index rises on the second day

a) By referring to the four outcomes, show that $(A \cap B) \cup (\bar{A} \cap B) = B$
b) By referring to the four outcomes, show that $A \cup (\bar{A} \cap B) = (A \cup S)$

7. The manager of an investment fund is considering whether or not to invest in an enterprise. In this environment of uncertainty, she estimates that the probabilities for the rate of return, which is a random variable, are the following:

Rate of return	Less than minus 10 %	Minus 10 % to 0 %	0 % to + 10 %	+ 10 % to + 20 %	+ 20 %
Probability	0.04	0.14	0.28	0.33	0.21

Consider the following two events :

A : the rate of return exceeds 10 %
B : the rate of return is negative

a) Find the probability that event A occurs.
b) Find the probability that event B occurs.
c) Describe the event of the complement of event A.
d) Find the probability of realizing that particular event.
e) Describe the event that is the intersection of events A and B.
f) Find the probability of realizing that particular event.
g) Describe the event that is the union of events A and B.
h) Find the probability of realizing that particular event.
i) Are the events A and B mutually exclusive? Explain why or why not.
j) Are the events of A and B collectively exhaustive? Explain why or why not.

8. For this question, consider the two events A and B.

a) The probability of realizing A is 0.40, the probability of realizing B is 0.45, and the probability of realizing either A or B is 0.85. What is the joint probability of these two events occurring?

b) Now suppose that the probability of realizing A is 0.80, while the probability of realizing B is 0.10, and the probability of both of them occurring simultaneously is 0.08. What is the probability of A conditioned on B? Compare it to the marginal probability of A, and explain the difference.

c) Given the values of the probabilities in part b), determine whether these two events are stochastically independent. There are two ways of demonstrating this, and you should show both of them. Explain your responses.

B. Written Questions

To answer the questions below, you need only recall (and write coherently about) what I said in class and wrote explicitly on the chalk board.

9. Identify and explain the significance of a random sample.

10. Identify and explain the significance of a scatter plot.

11. What is the difference between cross-sectional data and time-series data?

12. What is the relationship between the relative frequencies and the probabilities of the various realizations of a random variable?

13. In about one paragraph, write a very broad outline of the essence of probability and statistics. I suggest that you consider the following checklist: population, uncertainty, random variable, probabilities, sample, and statistical analysis. The idea is to link them together.