

MAT 1341A Test 1, 2013

5-October, 2013.

Instructor: Barry Jessup.

Family Name: _____

First Name: _____

Student number: _____

Enter your multiple choice
 responses here →

For the marker's use only →

1	
2	
3	
4	
5	
6 [Bonus]	
Total	

PLEASE READ THESE INSTRUCTIONS CAREFULLY.

1. You have 80 minutes to complete this exam. Read each question carefully.
2. This is a closed book exam, and no notes of any kind are allowed. The use of calculators, communication devices, or any image or text storage device is not permitted.
3. Questions 1 to 3 are multiple choice. These questions are worth 1 points each and no part marks will be given. Please record your answers in the space provided above.
4. Questions 4 and 5 require a complete solution, and are worth 6 points each, **so spend your time accordingly.**
5. Question 6 is a bonus question and is worth 2 points. To earn points here will be *much* more difficult than in questions 1-5.
6. **The correct answer in questions 4–6 requires justification written legibly and logically: you must convince the marker that you know why your solution is correct. You must answer these questions in the space provided.** Use the backs of pages if necessary.
7. Where it is possible to check your work, do so.
8. Good luck! Bonne chance!

1. Suppose $\{u, v, w\}$ is a set of vectors in a vector space V . Which of the following statements are equivalent to

“ $\{u, v, w\}$ is linearly independent.”

- I. None of the vectors u, v or w is a multiple of any other single vector in $\{u, v, w\}$.
 - II. If a, b, c are scalars, then $au + bv + cw = 0$ implies $a = b = c = 0$.
 - III. Both $\{u, v\}$ and $\{v, w\}$ are linearly independent.
 - IV. If $a = b = c = 0$, then $au + bv + cw = 0$.
 - V. None of the vectors u, v or w is a linear combination of the other vectors in $\{u, v, w\}$.
- A. I & II
 - B. I & III
 - C. II & III
 - D. II & V
 - E. II & IV
 - F. III & IV

2. Let $X = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \in \mathbf{M}_{22} \mid a - d = 0 \right\}$. Which of the following is a spanning set for X ?

- A. $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$
- B. $\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \right\}$
- C. $\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$
- D. $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$
- E. $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$
- F. $\left\{ \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right\}$

3. Which of the following are subspaces of \mathbf{M}_{22} ?

$$U = \left\{ \begin{bmatrix} x & y \\ y & z \end{bmatrix} \in \mathbf{M}_{22} \mid x, y, z \in \mathbf{R} \right\}$$

$$V = \left\{ \begin{bmatrix} 0 & y \\ -y & 0 \end{bmatrix} \in \mathbf{M}_{22} \mid y \in \mathbf{R} \right\}$$

$$W = \left\{ \begin{bmatrix} x & y \\ z & w \end{bmatrix} \in \mathbf{M}_{22} \mid xw - zy = 0 \right\}$$

$$S = \left\{ \begin{bmatrix} x & y \\ z & -x \end{bmatrix} \in \mathbf{M}_{22} \mid x, y, z \in \mathbf{R} \right\}$$

- A. Only U and V
- B. Only U and W
- C. Only U, V and W
- D. Only V, W and S
- E. Only W and S
- F. Only U, V and S

4. Let $n_0 = (1, 0, 1) \in \mathbf{R}^3$ and define

$$U = \{v \in \mathbf{R}^3 \mid \|v - n_0\| = \|v + n_0\|\}.$$

a) If $v \in \mathbf{R}^3$, show by expanding both sides using the dot product that

$$\|v - n_0\|^2 = \|v + n_0\|^2 \iff v \cdot n_0 = 0.$$

(★) Now we know that $U = \{v \in \mathbf{R}^3 \mid v \cdot n_0 = 0\}$, and thus U is a subspace of \mathbf{R}^3 .

b) Give a complete geometric description of U . (Hint: use (★)!)

c) Find a spanning set for U .

d) Is your spanning set in (c) linearly independent?

5. Let $\mathbf{F}[0, \pi] = \{f \mid f : [0, \pi] \rightarrow \mathbf{R}\}$ be the vector space of real-valued functions defined on $[0, \pi]$. Define four functions in $\mathbf{F}([0, \pi])$ by

$$f(x) = 1, \quad g(x) = \cos 2x, \quad h(x) = \cos x, \quad \text{and} \quad k(x) = \sin^2 x, \quad \forall x \in [0, \pi],$$

and let $W = \text{span}\{f, g\}$.

- a) Show that $\{f, g\}$ is linearly independent.
- b) Show that $h \notin W$.
- c) Use trigonometric identities to show that $k \in W$. (*Hint:* $\cos 2x = \cos(x + x)$.)

6. [Bonus] Let U and V be subspaces of a vector space W such that the subset

$$U \cup V = \{w \in W \mid w \in U \text{ or } w \in V\}$$

is also a subspace of W .

Prove carefully that either $U \cup V = U$ or $U \cup V = V$.

(You may find the following fact useful: If U is not a subset of V and V is not a subset of U , then there exists $u \in U$ with $u \notin V$ and $v \in V$ with $v \notin U$.)

