Carleton University

Dept. of Systems and Computer Engineering

Systems and Simulations—SYSC 3600 Winter 2015

Homework #1

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1. Find the Laplace transform of $e^{-\alpha t}\sin(\omega_0 t+\theta)$ and $e^{-\alpha t}\cos(\omega_0 t+\theta)$. What is the abscissa of convergence in each case?
2. Suppose that $\alpha=2$, ω_0=5 rad/second and $\theta=\pi/3$. Evaluate $\mathcal{L}\{e^{-\alpha t}\sin(\omega_0 t+\theta)\}$ at $s=2+j2$, $s=j$ and $s=-5+j$.
3. Show that $\mathcal{L}\{tf(t)\}=-\frac{d}{ds}F(s)$. Repeat for $\mathcal{L}\{t^2 f(t)\}$.
4. Given
$$F(s)=\frac{5(s+4)}{s(s+1)(s+2)}$$
   (a) Determine $f(\infty)$ without finding the inverse transform.
   (b) What is $f(0)$?
5. It is known that $\mathcal{L}\{e^{-t^2/2}\}=\sqrt{2\pi}e^{s^2/2}Q(-s)$, where $Q(\cdot)$ is the standard Q-function defined as $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-r^2/2}dr$.
   (a) What is the abscissa of convergence in this case?
   (b) Use the time-differentiation property to find the Laplace transform of $te^{-t^2/2}$. What is the abscissa of convergence in this case?