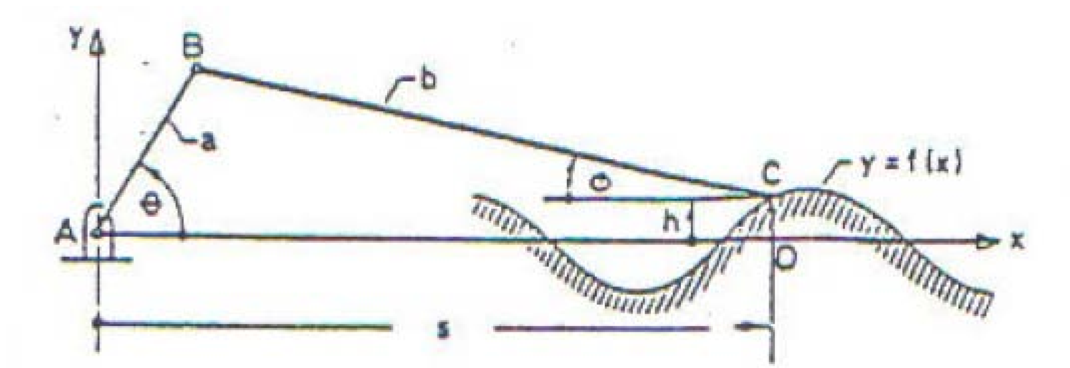
MCG 3130 **DGD 1: POSITION ANALYSIS**

Board Problem



Problem 1

If $\boldsymbol{\theta}$ is the input variable for the system as shown in the above figure and ***h = f (s)*** is a given smooth function of ***s***

a) Find the explicit expressions for corrections $\Delta\emptyset$ and $\Delta\boldsymbol{s}$ given by the Newton-Raphson Method for refining intial estimates of $\emptyset$ and $\boldsymbol{s}$.

b) Find $\emptyset$ and $\boldsymbol{s}$ for $\boldsymbol{\theta = 60^0}$ **and** $\boldsymbol{90^0}$ using **a = 4 inch, b=12 inch,** when**,**

f(x)= Cos (s), where ***s*** is in radians**.**

SOLN:

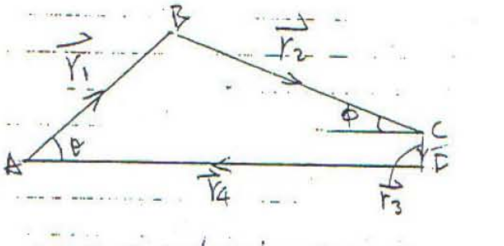
a) Loop closure equation of ABCDE

$\vec{r_1} = a\cos\theta\,\vec{i} + a\sin\theta\,\vec{j}$

$\vec{r_2} = b\cos\phi\,\vec{i} - b\sin\phi\,\vec{j}$

$\vec{r_3} = -h\,\vec{j}$

$\vec{r_4} = -s\,\vec{i}$



$$\vec{r_1} + \vec{r_2} + \vec{r_3} + \vec{r_4} = 0 \Rightarrow \begin{cases} a\cos\theta + b\cos\phi - s = 0 \\ a\sin\theta - b\sin\phi - h = 0 \end{cases}$$

we define two functions $f_1(s,\phi)$, $f_2(s,\phi)$

$$\Rightarrow \begin{cases} a\cos\theta + b\cos\phi - s = f_1(s,\phi) \\ a\sin\theta - b\sin\phi - h = f_2(s,\phi) \end{cases}$$

$$f_i(\phi + \Delta\phi, s + \Delta s) = 0 \quad i = 1, 2$$

$$\begin{cases} \frac{\partial f_1}{\partial \phi}\Delta\phi + \frac{\partial f_1}{\partial s}\Delta s = -f_1 \quad \frac{\partial f_2}{\partial \phi}\Delta\phi + \frac{\partial f_2}{\partial s}\Delta s = -f_2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} -b\sin\phi & -1 \\ -b\cos\phi & -f'(s) \end{bmatrix} \begin{pmatrix} \Delta\phi \\ \Delta s \end{pmatrix} = \begin{pmatrix} -f_1 \\ -f_2 \end{pmatrix}$$

$$\Rightarrow \Delta s = \frac{f_2\sin\phi - f_1\cos\phi}{f'(s)\sin\phi - \cos\phi} = \frac{(a\sin\theta - b\sin\phi - h)\sin\phi - (a\cos\theta - b\cos\phi - s)\cos\phi}{f'(s)\sin\phi - \cos\phi}$$

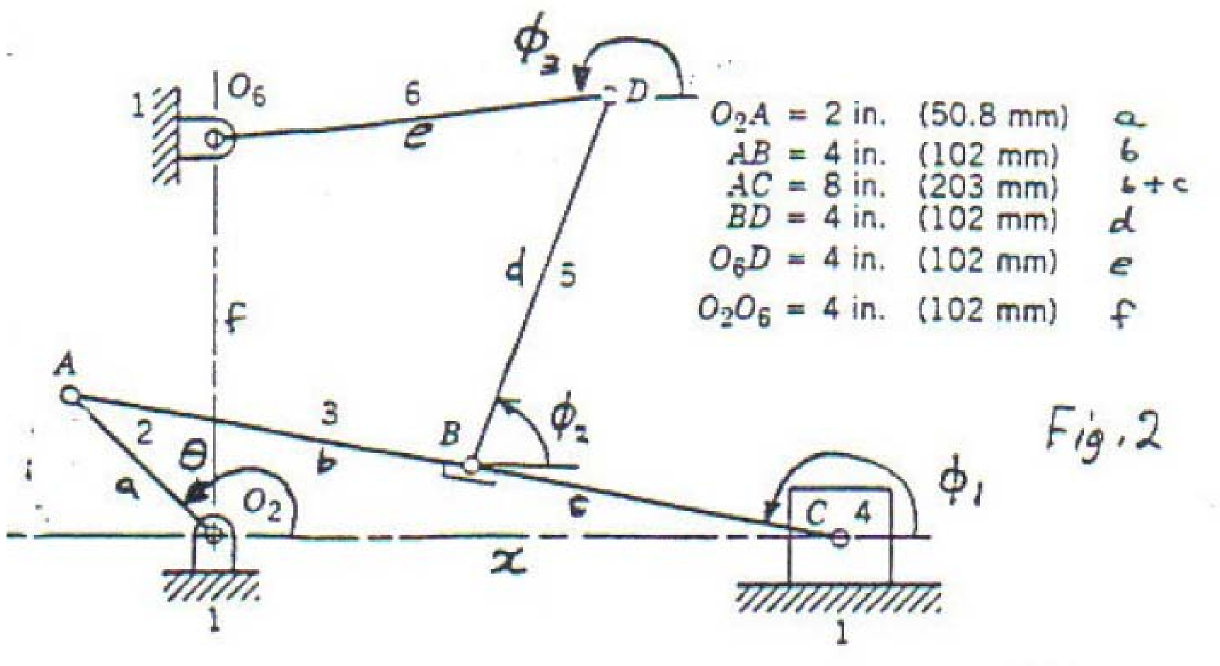
$$\Delta\phi = \frac{f_1 f'(s) - f_2}{b(f'(s)\sin\phi - \cos\phi)} = \frac{(a\cos\theta - b\cos\phi - s)f'(s) - (a\sin\theta - b\sin\phi - h)}{b(f'(s)\sin\phi - \cos\phi)}$$

b) For $\theta = 60°$ assume trial values $s = 42$ $\phi = \frac{2}{6}$ $h = \cos s = \cos 12 = 1$ $a = 4$ $b = 12$

Iterate using $\phi_{new} = \Delta\phi + \phi_{old}$ $s_{new} = \Delta s + s_{old}$ $\Rightarrow \phi = 0.250$ rad $s = 13.63$ in

For $\theta = 90°$ $a = 4$ $b = 12$ assume $s_0 = 32$ $\phi_0 = 2/3 \Rightarrow \phi = 0.30027$ rad $s \doteq 11.463$ in

MCG 3130 - DGD1 **Student's Assigned Problem**



Problem 2: **(10 points)**

a) Write the loop closure equations for **each independent loop** shown in the above figure 2.

b) Explain the procedure for finding $\boldsymbol{x}$, $\emptyset_1$, $\emptyset_2$, $\emptyset_3$ for a **given** value of angle $\boldsymbol{\theta}$.

SOLN:

a) $\vec{r}_1 = a\cos\theta\,\vec{i} + a\sin\theta\,\vec{j}$

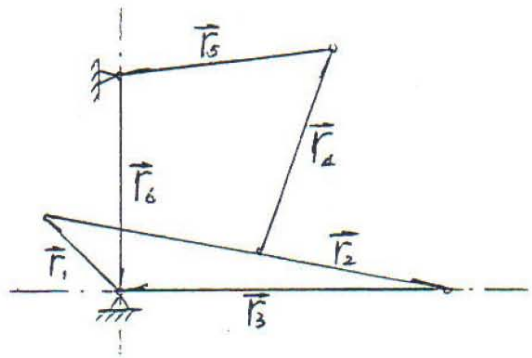
$\vec{r}_2 = -b\cos\varphi_1\,\vec{i} - b\sin\varphi_1\,\vec{j}$

$\vec{r}_3 = -x\,\vec{i}$

$\vec{r}_4 = d\cos\varphi_2\,\vec{i} + d\sin\varphi_2\,\vec{j}$

$\vec{r}_5 = e\cos\varphi_3\,\vec{i} + e\sin\varphi_3\,\vec{j}$

$\vec{r}_6 = -f\,\vec{j}$



Loop closure equations for O_2ACO_2

$$\vec{r}_1 + \vec{r}_2 + \vec{r}_3 = 0 \Rightarrow \begin{cases} b\cos\varphi_1 - x = a\cos\theta \\ b\sin\varphi_1 = a\sin\theta \end{cases} \qquad (1)$$

Loop closure equations for $O_2ABDO_6O_2$

$$\vec{r}_1 + \vec{r}_2/2 + \vec{r}_4 + \vec{r}_5 + \vec{r}_6 = 0$$

$$\text{or} \quad \begin{cases} d\cos\varphi_2 + e\cos\varphi_3 = \frac{b}{2}\cos\varphi_1 - a\cos\theta \\ d\sin\varphi_2 + e\sin\varphi_3 = f + \frac{b}{2}\sin\varphi_1 - a\sin\theta \end{cases} \qquad (2)$$

b) Solving loop closure equations (1), one obtains:

$$\begin{cases} \varphi_1 = \arcsin\left(\frac{a}{b}\sin\theta\right) \\ x = b\cos\varphi_1 - a\cos\theta \end{cases}$$

Using the Newton-Raphson method to solve equations (2), one will obtain a set of approximate solution of φ_2 and φ_3.