ENGR 391 – SAMPLE MIDTERM TEST

SOLUTIONS – 2011/2

1 a) we have $f(x) = x - x^{1/3} - 2 = 0$

Through incremental search, keeping the increment as 1, ($\because |a-b| = 1$)

x	0	1	2	3	4
$f(x)$	-2	-2	-1.2599	-0.4422	0.4126

$\therefore a = 3, \quad b = 4$

1 b)

a_i	b_i	$\frac{a_i+b_i}{2}$	Sign $f(a_i)$	Sign $f(b_i)$	Sign $f\left(\frac{a_i+b_i}{2}\right)$
3	4	3.5	−	+	−
3.5	4.0	3.75	−	+	+
3.5	3.75	3.625	−	+	+

2). $[A] = [L][U]$

$$\begin{bmatrix} -1 & 1 & -4 \\ 2 & 2 & 0 \\ 3 & 3 & 2 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

Taking the product L·U and equating to the corresponding element in A, we get

$l_{11} = -1$; $l_{11} u_{12} = 1 \longrightarrow u_{12} = -1$;

$l_{11} u_{13} = -4 \rightarrow u_{13} = 4$

$l_{21} = 2$; $l_{21} u_{12} + l_{22} = 2 \longrightarrow l_{22} = 4$;

$l_{21} u_{13} + l_{22} u_{23} = 0 \longrightarrow u_{23} = -2$;

$l_{31} = 3$; $l_{31} u_{12} + l_{32} = 3 \longrightarrow l_{32} = 6$;

$l_{31} u_{13} + l_{32} u_{23} + l_{33} = 2 \longrightarrow l_{33} = 2$;

$$[L] \cdot [U] = \begin{bmatrix} -1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 6 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

3). $[A]\{x\} = [L][U]\{x\} = \{b\}$

Let $Ux = y$. Then $Ly = b$.

$$\begin{bmatrix} -1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 6 & 2 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 1 \\ 3 \end{Bmatrix}$$

With forward substitution

$$y_1 = -2, \quad y_2 = \frac{1+4}{4} = 1.25$$

$$y_3 = \frac{3+6-7.5}{2} = 0.75$$

$$\begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} -2 \\ 1.25 \\ 0.75 \end{Bmatrix}$$

With backward substitution

$$x_3 = 0.75$$

$$x_2 = 1.25 + 1.5 = 2.75$$

$$x_1 = -2 + 2.75 - 3 = -2.25$$

4). $\|A\|_\infty$ = Maximum row sum

$$\begin{bmatrix} -1 & 1 & -4 \\ 2 & 2 & 0 \\ 3 & 3 & 2 \end{bmatrix} \quad \begin{aligned} \sum_{j=1}^{3} |a_{1j}| &= 6 \\ \sum_{j=1}^{3} |a_{2j}| &= 4 \\ \sum_{j=1}^{3} |a_{3j}| &= 8 \end{aligned} \qquad \therefore \|A\|_\infty = 8$$

$$[A]^{-1} = \begin{bmatrix} -0.6667 & 2.1667 & -1.3333 \\ 0.3333 & -0.8333 & 0.6667 \\ 0 & -0.7500 & 0.5000 \end{bmatrix} \begin{matrix} \rightarrow 4.1667 \\ \rightarrow 1.8333 \\ \rightarrow 1.2500 \end{matrix}$$

$$\|A^{-1}\|_\infty = 4.1667$$

$$\text{Cond}(A) = \|A\|_\infty \cdot \|A^{-1}\|_\infty$$

$$= 8 \times 4.1667 = \underline{\underline{33.3336}}$$

5). $f(x) = x^2 - \sin x = 0$

$f'(x) = 2x - \cos x$

With incremental search, first root is close to 0.8

x_i	$f(x_i)$	$f'(x_i)$	$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$
0.8	-0.0774	0.9031	0.8857
0.8857	0.0101	1.1387	0.8768
0.8768	0.0001	1.1140	0.8767

6. $f(x) = x^2 - \sin x = 0$

use 0.8 and 0.9 as the initial 2 points close to the expected root in order to calculate the slope $f'(x_i)$

In Secant Method $f'(x_i) = \dfrac{f(x_i) - f(x_{i-1})}{(x_i - x_{i-1})}$

and $x_{i+1} = x_i - \dfrac{f(x_i)}{f'(x_i)}$

x_{i-1}	x_i	$f(x_{i-1})$	$f(x_i)$	$f'(x_i)$	x_{i+1}
0.8	0.9	−0.0774	0.0267	0.8007	0.8667
0.9	0.8667	0.0267	−0.0110	1.0861	0.8768
0.8667	0.8768	−0.0110	0.0001	1.1140	0.8767