DGD 3: MCG 3130 Fall 2014 Spatial Mechanisms

Board Question

2. In the RSSC mechanism of Fig. 2 disk A is rotating at a constant counterclockwise angular speed of $10 rad/s$. Rod BC is connected to the disk and a collar by ball-and-socket joints. The collar radial dimension is small compared to the rod length.

(i) For the instant shown set up an equation set to determine the velocity $\mathbf{v}_B$ of the collar B along its guide, and the angular velocity ω_2 of the rod BC,

(ii) Assume the solution to part (i) is
$\omega_2 = 0.204\mathbf{i} - 0.612\mathbf{j} + 1.360\mathbf{k}$ rad/s; $\mathbf{v}_B = -0.334\mathbf{j}$ m/s
Determine then for this instant
(a) the velocity $\mathbf{v}_P$ of the mid-point P of the rod BC,
(b) the force $F_B\mathbf{j}$ required on the collar B to balance an applied force of $10\mathbf{i} + 20\mathbf{j} + 30\mathbf{k}$ N acting at P and a torque of $1000\mathbf{k}$ Nmm acting at A. Neglect gravity, friction and inertial effects.

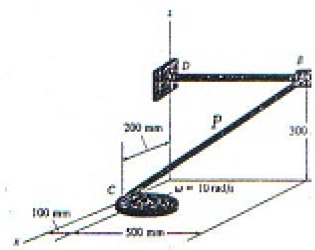
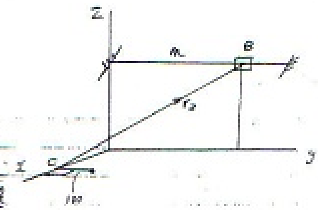


Fig. 2

SOLN:



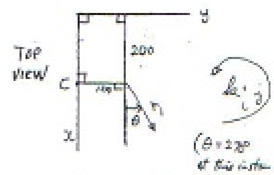
Define vectors

AC $\quad \underset{\sim}{r_1} = 100(\cos\theta\, \underset{\sim}{i} + \sin\theta\, \underset{\sim}{j}) = -100\underset{\sim}{j}$

CB $\quad r_2 = r_{2x}\underset{\sim}{i} + r_{2y}\underset{\sim}{j} + r_{2z}\underset{\sim}{k} = -200\underset{\sim}{i} + 600\underset{\sim}{j} + 300\underset{\sim}{k}$

BD $\quad \underset{\sim}{r_3} = -q\,\underset{\sim}{j}$

DA $\quad \underset{\sim}{r_0} = -300\underset{\sim}{k} + 200\underset{\sim}{i} + 100\underset{\sim}{j}$



(i) Loop closure $\quad \underset{\sim}{r_1} + \underset{\sim}{r_2} + \underset{\sim}{r_3} + \underset{\sim}{r_0} = \underset{\sim}{0}$

$$\underset{\sim}{\omega_1} \times \underset{\sim}{r_1} + \underset{\sim}{\omega_2} \times \underset{\sim}{r_2} + \dot{\underset{\sim}{r_3}} = 0$$

$$10\underset{\sim}{k} \times (-100\underset{\sim}{j}) + (\omega_{2x}\underset{\sim}{i} + \omega_{2y}\underset{\sim}{j} + \omega_{2z}\underset{\sim}{k}) \times (-200\underset{\sim}{i} + 600\underset{\sim}{j} + 300\underset{\sim}{k}) - \dot{q}\,\underset{\sim}{j} = \underset{\sim}{0}$$

$$1000\underset{\sim}{i} + (600\omega_{2x}\underset{\sim}{k} - 300\omega_{2x}\underset{\sim}{j} + 200\omega_{2y}\underset{\sim}{k} + 300\omega_{2y}\underset{\sim}{i} - 200\omega_{2z}\underset{\sim}{j} - 600\omega_{2z}\underset{\sim}{i} - \dot{q}\underset{\sim}{j} = \underset{\sim}{0}$$

i		$300\omega_{2y}$	$-600\omega_{2z}$	$=-1000$	in mm
j	$-300\omega_{2x}$		$-200\omega_{2z} - \dot{q}$	$=0$	
k	$600\omega_{2x}$	$+200\omega_{2y}$		$=0$	
8 also	$-200\omega_{2x}$	$+600\omega_{2y}$	$+300\omega_{2z}$	$=0$	from $\underset{\sim}{\omega_2}\cdot\underset{\sim}{r_2}=0$

Given Soln. 4 eqns for 4 unknowns $\quad \omega_{2x} = 0.204, \quad \omega_{2y} = -0.612, \quad \omega_{2z} = 1.36$ rad/s; $\underset{\sim}{v_B} = -0.334\underset{\sim}{j}$

(ii) (a) $\quad \underset{\sim}{v_P} = \underset{\sim}{v_C} + \underset{\sim}{\omega_2} \times \underset{\sim}{r_{CP}} = +10(.1)\underset{\sim}{i} + \frac{1}{2}(.204\underset{\sim}{i} - 0.612\underset{\sim}{j} + 1.36\underset{\sim}{k}) \times (-.2\underset{\sim}{i} + .6\underset{\sim}{j} + .3\underset{\sim}{k})$

$\qquad = 1\underset{\sim}{i} + [illegible]((-.184 - .816)\underset{\sim}{i} + (-.061 - .272)\underset{\sim}{j} + (.122 - .122)\underset{\sim}{k})$

4 $\qquad \therefore \boxed{\underset{\sim}{v_P} = 0.5\,\underset{\sim}{i} - 0.168\,\underset{\sim}{j}}$ m/s $\quad$ in m

3 $\quad$ (b) Principle of Virtual Work

$$\Sigma \underset{\sim}{F_m}\cdot\underset{\sim}{V_m} + \Sigma \underset{\sim}{T_n}\cdot\underset{\sim}{\omega_n} = 0$$

$$F_B\,\underset{\sim}{j}\cdot(-.334\underset{\sim}{j}) + (10\underset{\sim}{i} + 20\underset{\sim}{j} + 30\underset{\sim}{k})\cdot(0.5\underset{\sim}{i} - 0.168\underset{\sim}{j}) + 1\underset{\sim}{k}\cdot 10\underset{\sim}{k} = 0$$

$$-.334F_B + 5 - 3.36 + 10 = 0$$

$$\boxed{F_B = 34.9\,N}$$

DGD 3: MCG 3130 Fall 2013 **<u>Students Assigned Problem</u>:**

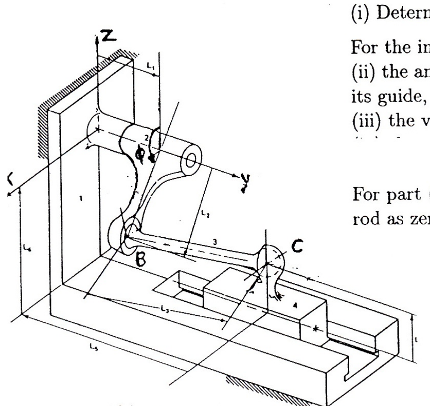


Fig 2

2. In the RSSP mechanism of Fig. 2 the connecting rod BC (link 3) is attached to a crank and a slider by ball and socket (spherical) joints. The crank is rotating CCW at a constant angular speed of 1 rad/s about the y axis. The constant link and offset dimensions L_1, L_2, L_3, L_4, and L_6 are respectively 1.0, 3.0, 9.0, 1.0, and 4.0cm. The diagram is not to scale.

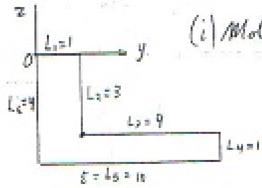
(i) Determine the mobility of the mechanism.

For the instant when $\phi = 180^o$, i. e. when link 2 is vertical down, determine
(ii) the angular velocity ω_3 of the connecting rod, and the velocity $\mathbf{v}_4$ of the slider (link 4) along its guide,
(iii) the velocity $\mathbf{v}_p$ of the mid-point P of the connecting rod,

For part (ii) above take the component of the angular velocity of the connecting rod along the rod as zero.

Solution:

Soln:



(i) Mobility $M = 6(n-1) - 5f_1 - 3f_3 = 18 - 10 - 6 = \underline{2}$

removable ϕ, and spin about con rod axis

Diagram for instant when $\phi = 180°$

$s = 10$

$\underset{\sim}{r_3} = 0\,\underline{i} + 9\,\underline{j} + 0\,\underline{k}$

$L_1\,\underline{j} + \underline{r_2} + \underline{r_3} - L_4\,\underline{k} - L_5\,\underline{j} + L_6\,\underline{k} = 0$

(ii) Loop closure equation $L_1\underline{j} + (L_2\cos\phi\,\underline{k} + L_2\sin\phi\,\underline{j}) + (r_{3x}\underline{i} + r_{3y}\underline{j} + r_{3z}\underline{k}) - L_4\underline{k} - s\underline{j} + L_6\underline{k} = \underline{0}$

Diff wrt time $[\dot{1}\,\underline{j} \times (-3\,\underline{k})] + [(\omega_{3x}\underline{i} + \omega_{3y}\underline{j} + \omega_{3z}\underline{k}) \times (9\underline{j})] - \dot{s}\,\underline{j} = 0$

$\underline{i}$ $\quad -9\omega_{3z} \quad -3 \quad = 0$ $\qquad$ $\dot{s}$ (i, j)

$\underline{j}$ $\quad \dot{s} \quad = 0$ $\qquad \Rightarrow \dot{s} = 0$

$\underline{k}$ $\quad 9\omega_{3x} \quad = 0$

Also $\underset{\sim}{\omega_3} \cdot \underset{\sim}{r_3} = 0$ i.e. $\omega_{3y} = 0$

So $\underline{\omega_{3x} = 0 \quad \omega_{3y} = 0 \quad \omega_{3z} = -\frac{1}{3} \quad v_4 = \underline{0}}$

(iii) Velocity of midpoint of link 3

$\underline{v_P} = \underline{v_4} + \underset{\sim}{\omega_3} \times \underset{\sim}{r_{CP}} = \underline{0} + (-\frac{1}{3}\underline{k}) \times (-4.5\underline{j}) = \underline{\underline{-1.5\,\underline{i}}}$ cm/s