MAT 2355, Fall 2014
Assignment 5 (10 points)

Due Monday, December 2. 2:30pm.

Instructor: Mohammad Bardestani

Student Name:

Student Number:

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.
Signature ________________________

Important:
Late assignments will **not** be accepted; **nor** will unstapled assignments.

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Question 1– [3 points] Show that an affine transformation T is an isometry if and only if its linear part is an orthogonal matrix.

Question 2– [3 point] Show that 2×2 matrix A is orthogonal if and only if

$$A = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}, \qquad \text{or} \qquad A = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{pmatrix},$$

for some θ.

Question 3– Prove the following equalities.

a) (1 point) $\langle u \times v, w \times z\rangle = \langle u, w\rangle\langle v, z\rangle - \langle v, w\rangle\langle u, z\rangle$.

b) (1 point) $||u \times v||^2 = ||u||^2||v||^2 - \langle u, v\rangle^2$.

c) (2 points) $u \times (v \times w) + v \times (w \times u) + w \times (u \times v) = 0$. This identity is called the Jacobi identity

Remark: Distributivity, linearity and Jacobi identity show that the $\mathbb{R}^3$ vector space together with vector addition and the cross product forms a Lie algebra. This is very important concept in geometry.