MAT 2355, Fall 2014
Assignment 3 (10 points)

Due Monday, October 6. 2:30pm.

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Student Name:

Student Number:

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.
Signature ________________________

Important:
Late assignments will **not** be accepted; **nor** will unstapled assignments.

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Question 1– [3 points] Let $C \subseteq \mathbb{R}^2$ be the circle with center P and radius r. Prove that if $T : \mathbb{R}^2 \to \mathbb{R}^2$ is an isometry then $T(C)$ is the circle with center $T(P)$ and radius r.

Question 2– [1 point] Show that reflection in the x axis in $\mathbb{R}^2$ is given by the formula

$$T(x, y) = (x, -y).$$

Question 3– [1 point] Show that the following map is a reflection.

$$\begin{aligned} T : \mathbb{R}^2 &\to \mathbb{R}^2 \\ (x, y) &\mapsto (x + 2, y - \pi), \end{aligned}$$

Question 4– [5 points]

0.1 Introduction

A **group** is a set G together with an operation $\circ$ satisfying the following requirements:

(G1) for each pair x, y of elements of G, $x \circ y$ is an element of G (closure axiom);

(G2) for all elements x, y, z of G, $(x \circ y) \circ z = x \circ (y \circ z)$ (associativity axiom);

(G3) there is an element e in G such that for all g in G $e \circ g = g = g \circ e$ (identity axiom);

(G4) given an element g in G, there is an element $g^* \in G$ such that $g \circ g^* = e = g^* \circ g$ (inverse axiom).

Example: Let G be the set $\mathbb{Z}$ of integers and the operation $\circ$ be addition of numbers. Then $\mathbb{Z}$ satisfies the four axioms under the operation $+$ if we take the element e in (G3) to be the integer 0, and the inverse x^* in (G4) to be the negative $-x$.

Example: Let $M_n(\mathbb{R})$ denote the set of $n \times n$ matrices with real entries. This is a group under addition of matrices if we take e to be the zero $n \times n$ matrix (all entries 0) and the inverse of the matrix X to be $-X$.

Example: Let G consist of the following four 2×2 matrices:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}.$$

We claim that these form a group under matrix multiplication. The easiest way to justify this claim is to calculate the table of all possible products.

	I	A	B	C
I	I	A	B	C
A	A	I	C	B
B	B	C	I	A
C	C	B	A	I

The table checks closure because each entry is in the set G. We can also see that I is the identity element and that each of I, A, B and C is its own inverse. Thus the table checks each of the group axioms apart from the associativity axiom, (G2). However, matrix multiplication is well known to be associative, and so this axiom is automatically satisfied.

Definition 1. *A group G is* **abelian** *if for all x and y in G, $x \circ y = y \circ x$.*

Example: $\mathbb{Z}$ is an abelian group since for any $x, y \in \mathbb{Z}$, $x + y = y + x$.

Example: Considering the operation of multiplication on the set $\mathbb{Z}$, we see that the first three axioms are satisfied, provided we take e to be the number 1. However, the fourth axiom fails in two ways. In the first place, zero has no inverse: there is no integer 0^* such that $0 \circ 0^* = 1$. Secondly, although there is a number 2^* with $2 \circ 2^* = 1$ (take 2^* to be $1/2$), there is no integer satisfying this condition.

0.2 Question

Using the introduction show

1. Show that ISO($\mathbb{R}^2$) is a group, and find
   (a) The identity element.
   (b) Inverse of Ω_ℓ.
   (c) Inverse of translation T_v.
2. The set of all translations of $\mathbb{R}^2$ is denoted by TRANS($\mathbb{R}^2$). Show that TRANS(R^2) is an abelian group.