

MAT 2355, Fall 2014  
Assignment 2 (10 points)

**Due Monday, September 29. 2:30pm.**

Instructor: Mohammad Bardestani

Student Name:

Student Number:

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature \_\_\_\_\_

Important:

Late assignments will **not** be accepted; **nor** will unstapled assignments.



**Question 1**– [2 points] Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$  be a linear transformation that preserves length; that is,

$\|T(v)\| = \|v\|$  for all  $v \in \mathbb{R}^n$ . Prove that for any  $v, w \in \mathbb{R}^n$  we have

$$\langle T(v), T(w) \rangle = \langle v, w \rangle.$$

(Hint: Use Question 2 of the assignment 1.)

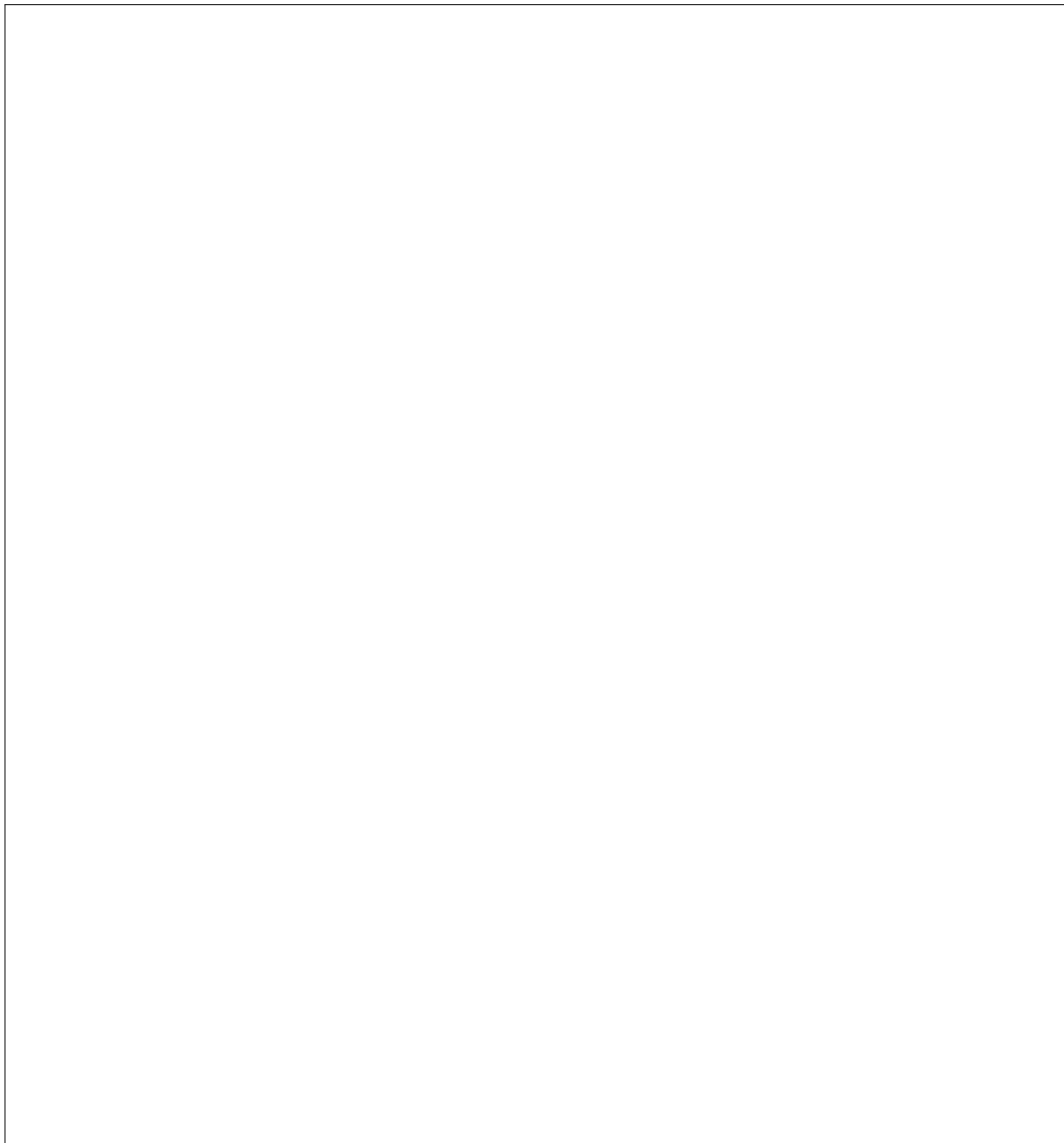
**Question 2**– [2 points] Let  $\{v_1, \dots, v_n\} \in \mathbb{R}^n$  be an orthonormal basis. Prove that for any  $v \in \mathbb{R}^n$  we have

$$\|v\|^2 = \langle v, v_1 \rangle^2 + \dots + \langle v, v_n \rangle^2.$$

**Question 3**– [1 point] Let  $\ell = (0, 1) + [(2, 1)]$  be a line in  $\mathbb{R}^2$ . Find the intersection point of the line through  $X = (3, 4)$  perpendicular to  $\ell$ .

**Question 4**– [2 points] Let  $X$  be a point in  $\mathbb{R}^2$  and  $\ell$  a line. Let  $F$  be the foot of  $X$  on  $\ell$ , then show that  $F$  is the point of  $\ell$  nearest to  $X$ . In other words if  $Q$  is any point of  $\ell$  then

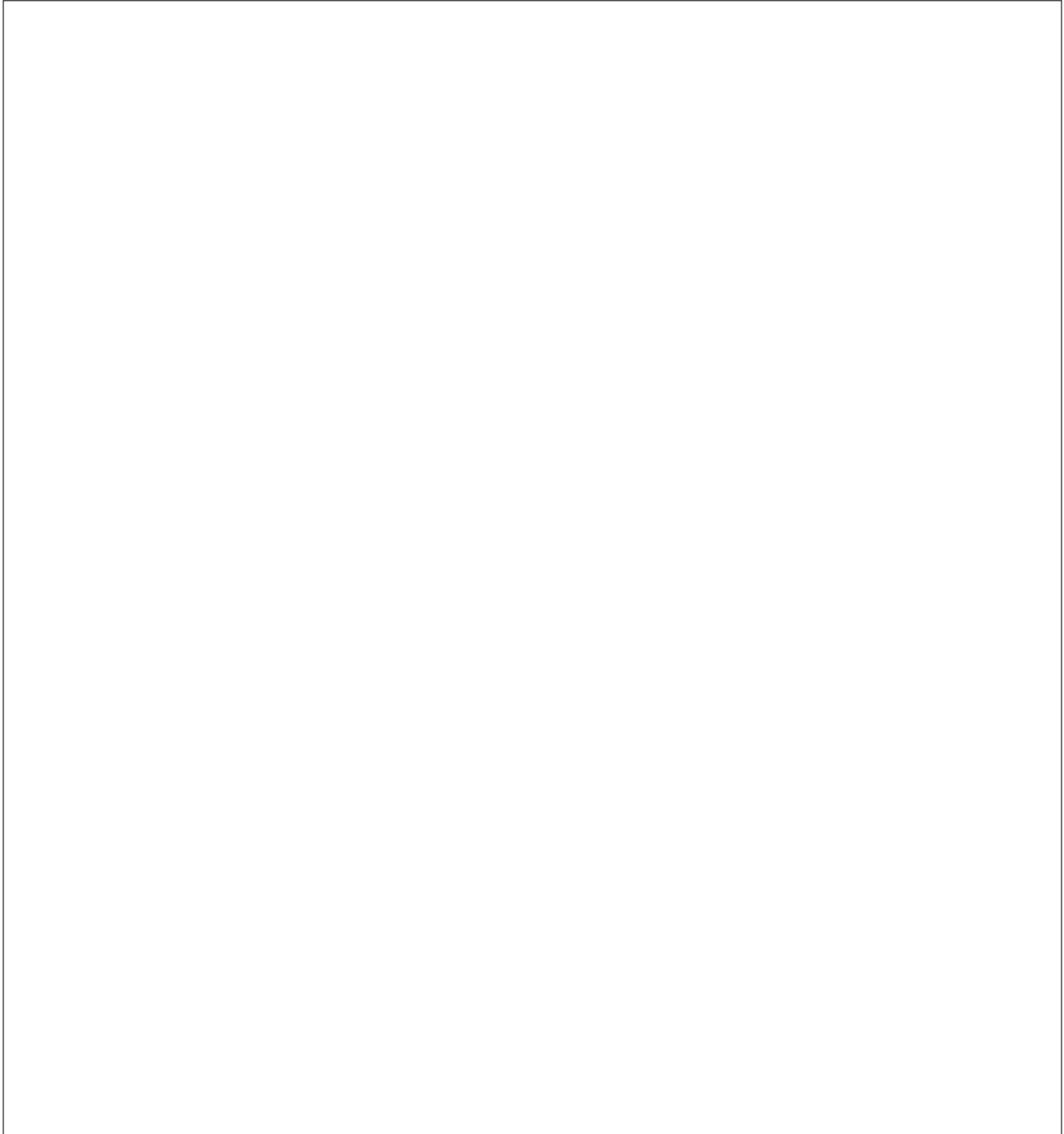
$$\|X - Q\| \geq \|X - F\|.$$



**Question 5**– [2 points] Let  $\ell_1$  and  $\ell_2$  be parallel lines. Let

$$\ell_3 := \{1/2(X_1 + X_2) : X_1 \in \ell_1, X_2 \in \ell_2\}.$$

Prove that  $\ell_3$  is a line parallel to  $\ell_1, \ell_2$ .



**Question 6**– [1 points] For given line  $\ell$ , let  $\Omega_\ell(X)$  be the reflection of point  $X$  with respect to line  $\ell$ . Show that  $\Omega_\ell(X) = X$  if and only if  $X \in \ell$ .