MAT 2355, Fall 2014
Assignment 2 (10 points)

Due Monday, September 29. 2:30pm.

Instructor: Mohammad Bardestani

Student Name:

Student Number:

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.
Signature ________________________

Important:
Late assignments will **not** be accepted; **nor** will unstapled assignments.

♠ ♠ ♠

Question 1– [2 points] Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation that preserves length; that is, $||T(v)|| = ||v||$ for all $v \in \mathbb{R}^n$. Prove that for any $v, w \in \mathbb{R}^n$ we have

$$\langle T(v), T(w) \rangle = \langle v, w \rangle.$$

(Hint: Use Question 2 of the assignment 1.)

Question 2– [2 points] Let $\{v_1, \ldots, v_n\} \in \mathbb{R}^n$ be an orthonormal basis. Prove that for any $v \in \mathbb{R}^n$ we have

$$||v||^2 = \langle v, v_1 \rangle^2 + \cdots + \langle v, v_n \rangle^2.$$

Question 3– [1 point] Let $\ell = (0,1) + [(2,1)]$ be a line in $\mathbb{R}^2$. Find the intersection point of the line through $X = (3,4)$ perpendicular to ℓ.

Question 4– [2 points] Let X be a point in $\mathbb{R}^2$ and ℓ a line. Let F be the foot of X on ℓ, then show that F is the point of ℓ nearest to X. In other words if Q is any point of ℓ then

$$||X - Q|| \geq ||X - F||.$$

Question 5– [2 points] Let ℓ_1 and ℓ_2 be parallel lines. Let

$$\ell_3 := \{1/2(X_1 + X_2) : X_1 \in \ell_1, \quad X_2 \in \ell_2\}.$$

Prove that ℓ_3 is a line parallel to ℓ_1, ℓ_2.

Question 6– [1 points] For given line ℓ, let $\Omega_\ell(X)$ be the reflection of point X with respect to line ℓ. Show that $\Omega_\ell(X) = X$ if and only if $X \in \ell$.