MAT 2355, Fall 2014

Assignment 1 (10 points)

Due Monday, September 22. 2:30pm.

Instructor: Mohammad Bardestani

Student Name:

Student Number:

By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.
Signature ________________________

Important:
Late assignments will **not** be accepted; **nor** will unstapled assignments.

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Question 1– [2 points] Let P, Q be two points in $\mathbb{R}^2$. If $0 < t < 1$ and $X = (1-t)P + tQ$, show that

$$\frac{d(P,X)}{d(X,Q)} = \frac{t}{1-t}.$$

Use this to find the point X that divides the segment PQ in the ration $r : s$.

Question 2– [2 points] For $v, w \in \mathbb{R}^n$ show that

$$\langle v, w \rangle = \frac{1}{2}\left(||v||^2 + ||w||^2 - ||v - w||^2\right)$$

Question 3– [2 points] Let $X \in \mathbb{R}^2$ be a point and let ℓ be a line in plane. Prove that there exists a unique line $\mathfrak{m}$ through X perpendicular to ℓ.

Question 4– [1 points] Let $v_1, \ldots, v_k \in \mathbb{R}^n$. Show that if $v \perp v_i$, for $1 \leq i \leq k$, the $v \in W^{\perp}$, where W is the subspace of $\mathbb{R}^n$ generated by $v_1, \ldots, v_k$.

Question 5– [2 points] Let $v_1 = (3, 1, 1)$, $v_2 = (-1, 2, 1)$ and $v_3 = (-1/2, -2, 7/2)$ be three vectors in $\mathbb{R}^3$.

Show that $\{v_1, v_2, v_3\}$ is an orthogonal basis for $\mathbb{R}^3$. Also express $y = (6, 1, -8)$ as a linear combination of v_1, v_2, v_3.

Question 6– [1 points] Given points A and B, find, with a Euclidean compass alone, the midpoint M of segment AB. Prove your method.