PLEASE READ THESE INSTRUCTIONS CAREFULLY!

1. Read each question carefully, and **answer all questions in the space provided after each question.** For questions 4 to 7, you may use the backs of pages if necessary, but be sure to indicate to the marker that you have done this.
2. Questions 1 to 3 are multiple choice. These questions are worth 1 point each and no part marks will be given. Please record your answers 1-3 in the space provided below.
3. Questions 4 and 5 are worth points each, Question 6 is worth , and part marks can be earned. **The correct answers here require justification written legibly and logically: you must convince the marker that you know why your solution is correct.**
4. Where it is possible to check your work, do so.
5. Good luck! Bonne chance!

Name and Student ID number: ______________________________

Signature and date: ______________________________

Question	1	2	3	4	5	6	Total
Answer				X	X	X	
Grade							

1. Which of the following statements is true for the linear system (in 5 variables)?

$$\begin{array}{rcrcrcrcrl} x_1 & - & x_2 & & & + & 2x_4 & + & 2x_5 & = 0 \\ & & & & x_3 & + & 6x_4 & - & x_5 & = 3 \end{array}$$

   A. The system has no solutions

   B. $(-3r + 2s + 2t, r, 6s - t, s, t)$ is a solution for any values of r, s and t

   C. $(-9, 2, 1, 1, 4)$ is the unique (only) solution of the system

   D. $(-3r + 2s + 2t, 3, 6s + t, s, t)$ is a solution for any values of s and t

   E. $(3r - 2s - 2t, 3, -6s + t, s, t)$ is a solution for any values of r, s and t

   F. $(3r - 2s - 2t, r, 3 - 6s + t, s, t)$ is a solution for any values of r, s and t

2. If $\{u, v, w\}$ is a linearly **dependent** set in vector space V, and that $\{v, w\}$ is linearly **independent**. Which of the following statements is **ALWAYS** true?

   A. $\{u, w\}$ is linearly independent.
   B. $\{v, u\}$ is linearly independent.

   C. $u \notin \text{span}\{v, w\}$.

   D. $u \in \text{span}\{v, w\}$.

   E. $v \in \text{span}\{u, w\}$.

   F. None of the above are true.

3. If a subspace X of $\mathbb{R}^{20}$ can be spanned by 14 vectors and contains a linearly independent set with 9 vectors, then any basis of X must have:

   A. At most 9 vectors.

   B. At least 10 vectors and at most 14 vectors.

   C. At least 9 vectors.

   D. At least 9 vectors and at most 13 vectors.

   E. At least 9 vectors and at most 14 vectors.

   F. None of the above is true.

4. Recall the vector space $\mathbb{P}_3 = \{a + bx + cx^2 + dx^3 \mid a, b, c \in \mathbb{R}\}$ of polynomial functions of degree at most 3, and define

$$V = \{p \in \mathbb{P}_3 \mid p(-3) = 0\}.$$

(a) Show that $V = \text{span}\{x^3 + 3x^2, x^2 + 3x, x + 3\}$. *(Hint: recall the Factor Theorem: if p is any polynomial and $p(a) = 0$ for some $a \in \mathbb{R}$, then $p(x) = (x - a)q$ for some polynomial q with degree one less than that of p.)*

(b) Explain why V is a subspace of $\mathbb{P}_3$ *without using the subspace test.*

(c) Find a basis for V.

(d) Find $\dim V$.

5. Let $S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathrm{M}_{22} \mid a + d = 0 \right\}$.

   (a) Find a basis for S and hence determine $\dim S$.

(b) Does your basis for S span M_{22}? Why or why not?

(c) Extend your basis in (a) to a basis of M_{22}.

6. State whether each of the following statements is (always) true, or is (possibly) false, in the box after the statement.

- If you say the statement may be false, you must give an explicit example - with numbers!
- If you say the statement is always true, you must give a clear explanation.

(a) If U and V are subspaces of $\mathbb{R}^2$, then their intersection

$$U \setminus V = \{v \in \mathbb{R}^2 \mid v \in U \text{ and } v \notin V\}$$

is also a subspace.

ANSWER

(b) If V is a vector space and $\{v_1, v_2\} \subset V$ is linearly dependent, then v_1 and v_2 are scalar multiples of each other.

ANSWER

(c) If u_1, u_2, u_3, and u_4 are linearly independent vectors in a vector space V, then $\dim U = 4$.

ANSWER

(d) If U is a 4-dimensional subspace of $\mathbb{R}^6$, it is possible to find a spanning set for U that contains 3 vectors.

ANSWER

(You may use this page for rough work or solutions that did not fit on previous pages.)