

University of Ottawa

School of Management

ADM 2303: STATISTICS FOR MANAGEMENT I

FINAL EXAMINATION

7.00 pm. December 19, 2002

Time allowed: 3 hours.

Closed book exam except that one page of notes (8.5 x 11 inches, written on both sides, without any additional paper stuck on) is allowed. Calculators are allowed.

Last Name (PRINT)

First Name (PRINT)

Student Number

Section: Circle one: A B C D E

Professor Wright: sections A,B

Professor Manoussi: sections C,D,E.

Signature

WRITE ALL ANSWERS (INCLUDING BRIEF EXPLANATIONS) ON THE ANSWER SHEET. The exam question sheets will **not** be marked, though space is provided here for rough work. The question sheets **must** be deposited in the box provided at the end of the exam. Half the marks are for methodology, half are for a correct numerical answer. The total number of marks on this exam is 88. The mark will be scaled to represent 50% of the course mark.

1. After analyzing several months of sales data, the owner of an appliance store produces the following joint probability distribution of the number of refrigerators and stoves sold daily.

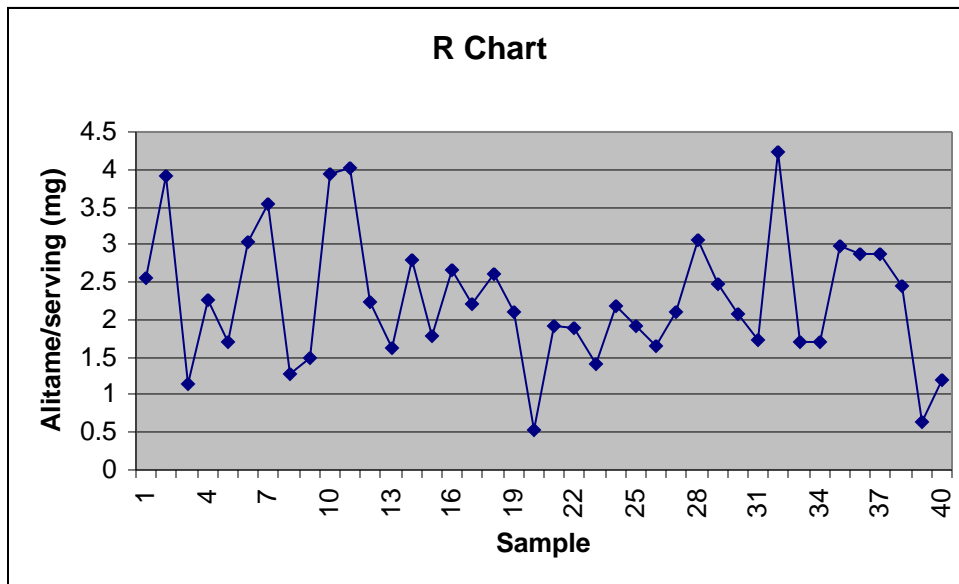
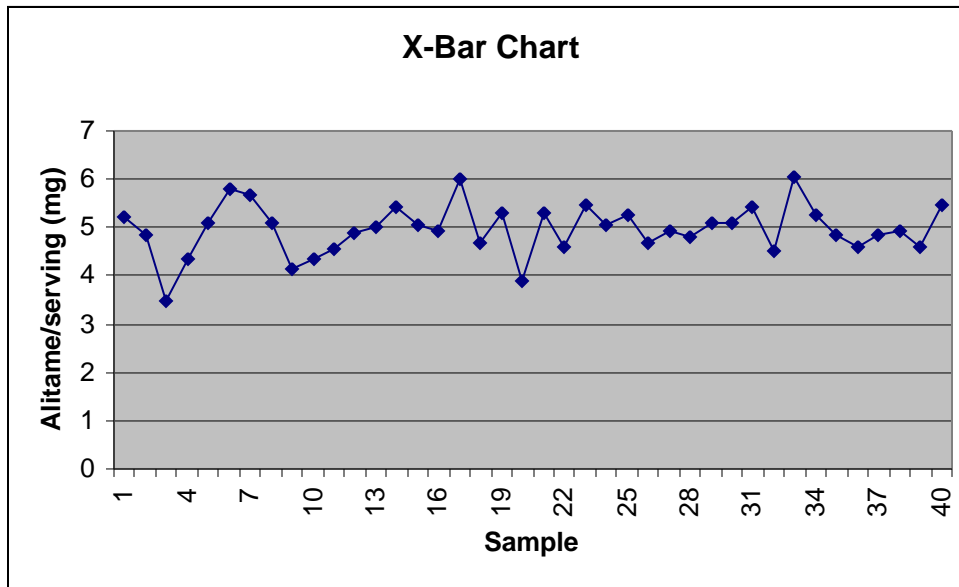
Stoves	Refrigerators		
	0	1	2
0	.08	.14	.12
1	.09	.17	.13
2	.05	.18	.04

For instance, the probability of selling 2 stoves and 1 refrigerator on any day is 0.18.

- a. (3) Find the probability of the number of refrigerators sold daily being (i) 0, (ii) 1 and (iii) 2.
 - b. (3) Find the probability of the number of stoves sold daily being (i) 0, (ii) 1 and (iii) 2.
 - c. (2) Compute the mean and variance of number of refrigerators sold daily.
 - d. (2) Compute the mean and variance of number of stoves sold daily.
 - e. (4) Compute the covariance and the coefficient of correlation between the number of refrigerators and the number of stoves sold daily.
 - f. (4) Compute the mean and variance of the total number of these appliances (refrigerators plus stoves) sold daily
2. (8) Your favorite team is in a competition in which they play a number of games in order to win a championship. Prior to the start of the competition you have assigned a probability of 60% that they will win the championship. Past records indicate that when teams win the championship, they win the first game of the competition 70% of the time. When they lose the championship, they win the first game 25% of the time. The first game is over, your team has lost. Now, what is the probability that they will win the championship?
3. In order to comply with the Kyoto Protocol, federal government regulations specify a limit on methane emissions from landfills. Each day, city authorities measure the amount of gas generated by landfills and its methane content. The federal regulations require that on at least 94% of days the amount of methane should not exceed 55% of the total amount of gas.
- a. (4) City A reports that the percentage of methane generated by its landfills is *normally* distributed with a mean of 45% and a standard deviation of 7%. Is City A complying with federal regulations? Show your reasons clearly.
 - b. (6) City B reports that the percentage of methane generated by its landfills is *symmetrically*

distributed with a mean of 40% and a standard deviation of 5%. Is City B complying with federal regulations? Show your reasons clearly.

4. A manufacturer of frozen deserts is planning to use alitame as an artificial sweetener. Trials have found that consumers prefer about 5 mg of alitame per serving. The production process is therefore set to add 5 mg of alitame per serving to the deserts, however it cannot guarantee to add exactly 5 mg and tests on the frozen deserts are conducted to analyse how much has actually been added. 5 frozen deserts are selected at random each hour for 40 hours. The alitame content per serving is summarized in the following graphs of the average (X-Bar) and range (R) of alitame in the 5 deserts in each sample.



n:	A ₂	D ₃	D ₄
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00

Over the 40 samples, the average of \bar{X} is 5.07 mg and the average of R is 2.31 mg.

(a) (4) Using the Table of Shewhart factors A_2 , D_3 , D_4 , calculate the upper and lower control limits for \bar{X} -Bar and R .

(b) (8) Apply 4 control rules to the \bar{X} -Bar and R charts. Indicate on the graphs on the answer sheet any areas where these rules fail. Clearly indicate which rule fails where.

(c) (4) What conclusions do you draw about the process for adding alitame to frozen deserts.

5. In West Africa, the yield of peanuts per hectare using traditional farming methods is normally distributed with a mean of 6 tons per hectare and a standard deviation of 1.5 tons per hectare.

a. (4) What is the probability that the yield from a randomly selected traditional farm is more than 7 tons per hectare?

b. (2) What is the probability that in random sample of five traditional farms, each of them has a peanut yield of more than 7 tons per hectare?

c. (8) The Agricultural Research Station in Zaria, Nigeria, has developed an improved fertilizer treatment, which is claimed to increase the yield of peanuts. The new treatment is tested on 5 randomly selected traditional farms and produces the following yields: 7.2, 6.8, 6.5, 7.3, 7.2 tons per hectare. Before recommending the new treatment for widespread use, the director of the research station wants to be 95% sure that it increases yields. You are the research station statistician. What advice do you give the director? Back up your advice with a statistical reasoning.

d. (6) Suppose that for the traditional farming methods, we know the mean and standard deviation yields are 6 and 1.5 tons per hectare respectively (as in the original statement of this question) but we do not know whether the distribution is Normal. Using the same data in (c) what advice do you give the research station director now? Back up your advice with statistical reasoning. NB the director does not like “approximate” advice.

6. Customers send short text messages over a cellular network. Messages are processed by a router in order to be sent to the correct destination. Messages arrive at the router at random, at an average rate of 1 message every 2 milliseconds.

a. (8) During 1 millisecond, the router can process a maximum of 4 messages. Any additional message arriving during that 1 millisecond are lost. The cellular company performance standards require that no more than 0.1% of messages should be lost. Is this router meeting those standards? Back up your answer with a statistical calculation.

b. (4) A message has just arrived. What is the probability of having to wait **at least** 4 milliseconds until the next message arrives.

c. (4) A message has just arrived. What is the probability of having to wait **exactly** 4 milliseconds until the next message arrives.

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(a) (3) $(p(0)=.22, p(1)= .49, p(2)= .29)$

(b) (3) $(p(0)=.34, p(1)= .39, p(2)= .27)$

(c) (2) Mean = $\mu=0*0.22+1*0.49+2*0.29=1.07,$
 $\sigma^2=(0.22*(0-1.07)^2+0.49*(1-1.07)^2+0.29*(2-1.07)^2)=.505$

(d) (2) Mean= $\mu=0*0.34+1*0.39+2*0.27=.93,$
 $\sigma^2==(0.34*(0-0.93)^2+0.39*(1-0.93)^2+0.27*(2-0.93)^2)=.605$

(e) (4) Cov(F,S) =
 $0.08*(0-1.07)*(0-0.93)+0.14*(1-1.07)*(0-0.93)+0.12*(2-1.07)*(0-0.93)+0.09*(0-1.07)*(1-0.93)+0.17*(1-1.07)*(1-0.93)+0.13*(2-1.07)*(1-0.93)+0.05*(0-1.07)*(2-0.93)+0.18*(1-1.07)*(2-0.93)+0.04*(2-1.07)*(2-0.93)$
 $=-0.0451,$

Corr coeff, $\rho = -0.0451/\text{SQRT}(0.6051*0.5051)=-0.0816$

(f) (4) $T=F+S$

Mean(T)=Mean(F)+Mean(S) = $1.07+0.93 = 2.00$

Var(T)=Var(F)+Var(S)+2Cov(F,S) = $0.505 + 0.605 - 2*0.0451 = 1.02$

Alternatively one could enumerate the probabilities of selling 0,1,2,3,...,6 appliances

2 (8) Priors $P(\text{win champ})=0.6$ $P(\text{lose champ})=0.4$

Condit:

$P(\text{Lose 1st game} | \text{win champ}) = 0.3; P(\text{Lose 1st game} | \text{lose champ}) = 0.75$

Joint: $P(\text{Lose 1st game} \& \text{win champ}) = 0.3*0.6 = 0.18$

$P(\text{Lose 1st game} \& \text{lose champ}) = 0.75*0.4 = 0.3$

Posterior: $P(\text{win champ} | \text{lose first game}) = 0.18/(0.18+0.3) = 0.375$

3.

a. (4) Using Normal Distribution, $z=(55-45)/7 = 1.429;$ $P=0.923.$ No. City A is only complying on 92.3% of days, not the required 94%

b. (6) We are not told the distribution, so must use Tchebysheff's Theorem. $1-1/n^2$ observations are within n standard deviations either side of the mean. For $n=3,$ 89% of observations are within 3 standard deviations, i.e. within the range 25% - 55%. Since the distribution is symmetric, 5.5% of observations are $> 55\%.$ City B is complying on 94.5% of days as required.

4.

(a) (4) For X-Bar: $UCL=5.07+0.58*2.31=6.41; LCL=5.07-0.58*2.31=3.73$

For R: $UCL=2.11*2.31=4.87; LCL = 0$

(b) (8) A point outside the control limits	Does not fail
9 points in a row above (below) the center line	Fails for R: points 19-27
6 points in a row increasing (decreasing)	Fails for X-Bar: points 10-15
14 points in a row alternating up/down	Fails for X-Bar: points 16-30

(c) (4) The process is operating inconsistently, is out of control.

5

(a) (4) $P(X > 7) = P((X - \mu_x) / \sigma_x > (7 - 6) / 1.5) = P(Z > 0.6667) = .5 - .2475 = .2525$

(b) (2) $P(X > 7) * P(X > 7) * P(X > 7) * P(X > 7) * P(X > 7) = .2525 * .2525 * .2525 * .2525 * .2525 = .00103$

(c) (8) $\mu_x = \mu = 6, \sigma_x = \sigma = 1.5 / \sqrt{5} = .671$

Mean of sample = $\frac{7}{5}$

$P(X > 7) = P((X - \mu_x) / \sigma_x > (7 - 6) / .671) = P(Z > 1.491) = .5 - .4320 = .0680$

$P(\text{increased yields}) = 1 - 0.0681 = 0.9319$. Not 95% sure.

Do not recommend widespread use of new fertilizer treatment.

(d) (6) Since population not Normal, need to use CLT, for which need large n.
Advice: Get a larger sample.

6.

a. (8) Poisson: $\lambda = 0.5$ msgs per msec.

$P(0) = e^{-(0.5)}$

$P(1) = 0.5 * e^{-(0.5)}$

$P(2) = 0.5^2 * e^{-(0.5)} / 2$

$P(3) = 0.5^3 * e^{-(0.5)} / 6$

$P(4) = 0.5^4 * e^{-(0.5)} / 24$

$P(>4) = 1 - 2.71828^{-(0.5)} [1 + 0.5 + 0.5^2/2 + 0.5^3/6 + 0.5^4/24] = 0.000172$

Yes: meeting standards. 0.017% of message are being lost.

b. (4) $P(> 4 \text{ msecs}) = 2.71828^{-(4*0.5)} = 0.1353$

c. (4) $P = 0$; The probability of a continuous distribution taking any exact value is zero.