UNIVERSITY OF WINDSOR
DEPARTMENT OF MATHEMATICS AND STATISTICS
Integral Calculus 62-141-01
Midterm Exam 1
Wednesday, June 5, 2013

Last name (PRINT): ______________

First name: ______________

Student No.: ___________

Section No.: ___________

Instructions :

- This test has 9 problems and a total of 7 pages, including this cover page. You have 80 minutes.
- Read carefully and answer all questions. Show all the work.
- No calculators, cell phones or other electronic devices are allowed.
- Work all problems in the space provided.
- You must give exact answers (and not decimal approximations).

Question	Points	Score
1	8	
2	8	
3	8	
4	8	
5	8	
6	8	
7	8	
8	16	
9	8	
Total:	80	

Evaluate the following integrals.

(8) 1. $\displaystyle\int \frac{e^x}{(e^x+2)(e^x-3)}\,dx$

$$= \int \frac{1}{(u+2)(u-3)}\,du$$

$$= \int \left[-\frac{1}{5(u+2)} + \frac{1}{5(u-3)} \right] du$$

$$= -\frac{1}{5}\ln|u+2| + \frac{1}{5}\ln|u-3| + C$$

$$= -\frac{1}{5}\ln|e^x+2| + \frac{1}{5}\ln|e^x-3| + C$$

Let $u = e^x$

$du = e^x\,dx$

$$\frac{1}{(u+2)(u-3)} = \frac{A}{u+2} + \frac{B}{u-3}$$

$$1 = A(u-3) + B(u+2)$$

$u=3 \Rightarrow 1 = 5B \Rightarrow B = \frac{1}{5}$

$u=-2 \Rightarrow 1 = A(-5) \Rightarrow A = -\frac{1}{5}$

(8) 2. $\displaystyle\int \frac{x^{55}}{\sqrt{1-x^{112}}}\,dx$

$$= \int \frac{x^{55}}{\sqrt{1-(x^{56})^2}}\,dx$$

$$= \int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= \sin^{-1}(u) + C = \sin^{-1}(x^{56}) + C$$

Let $u = x^{56}$

$du = 56x^{55}\,dx$

$\frac{1}{56}\,du = x^{55}\,dx$

(8) 3. $\int e^{3x} \cos 4x \, dx$

$+$	e^{3x}	$\cos 4x$
$-$	$3e^{3x}$	$\frac{1}{4}\sin 4x$
$+$	$9e^{3x}$	$\frac{-1}{16}\cos 4x$

$$I = \int e^{3x} \cos 4x \, dx$$

$$I = \frac{1}{4} e^{3x} \sin 4x + \frac{3}{16} e^{3x} \cos 4x - \frac{9}{16} I$$

$$\left(1 + \frac{9}{16}\right) I = \frac{1}{4} e^{3x} \sin 4x + \frac{3}{16} e^{3x} \cos 4x$$

$$\frac{25}{16} I = \frac{1}{4} e^{3x} \sin 4x + \frac{3}{16} e^{3x} \cos 4x$$

$$I = \frac{4}{25} e^{3x} \sin 4x + \frac{3}{25} e^{3x} \cos 4x + C$$

(8) 4. $\int \sin^{101} x \cos^3 x \, dx$

Let $u = \sin x$

$du = \cos x \, dx$

$$= \int \sin^{101} x \left(1 - \sin^2 x\right) \cos x \, dx$$

$$= \int u^{101} \left(1 - u^2\right) du$$

$$= \int \left(u^{101} - u^{103}\right) du$$

$$= \frac{u^{102}}{102} - \frac{u^{104}}{104} + C$$

$$= \frac{\sin^{102} x}{102} - \frac{\sin^{104} x}{104} + C$$

(8) 5. $\int \frac{\sqrt{x-36}}{x}\,dx$

Let $u^2 = x - 36$

$2u\,du = dx$

$$= \int \frac{\sqrt{u^2}}{u^2+36} \cdot 2u\,du$$

$$= 2\int \frac{u^2}{u^2+36}\,du$$

$$= 2\int \left[\frac{u^2+36}{u^2+36} - \frac{36}{u^2+36}\right] du$$

$$= 2\int \left[1 - \frac{36}{u^2+36}\right] du = 2u - 72 \cdot \frac{1}{6} \tan^{-1} \frac{u}{6} + C$$

$$= 2\sqrt{x-36} - 12\tan^{-1}\left(\frac{\sqrt{x-36}}{6}\right) + C$$

(8) 6. $\int \sqrt{9-x^2}\,dx$

Let $x = 3\sin\theta$. Then $dx = 3\cos\theta\,d\theta$

$$\int \sqrt{9-x^2}\,dx = \sqrt{9-9\sin^2\theta} \cdot 3\cos\theta\,d\theta$$

$$= \int 3\cos\theta \cdot 3\cos\theta\,d\theta$$

$$= 9\int \cos^2\theta\,d\theta$$

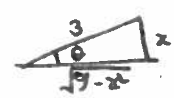
$$= \frac{9}{2}\int (1+\cos 2\theta)\,d\theta$$

$$= \frac{9}{2}\theta + \frac{9}{4}\sin 2\theta + C$$

$$= \frac{9}{2}\theta + \frac{9}{2}\sin\theta\cos\theta + C$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{9}{2} \cdot \frac{x}{3} \cdot \frac{\sqrt{9-x^2}}{3} + C$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) + \frac{x\sqrt{9-x^2}}{2} + C$$



$\sin\theta = \frac{x}{3}$

$\cos\theta = \frac{\sqrt{9-x^2}}{3}$

(8) 7. $\int \frac{x}{10+6x+x^2}dx$

$$= \int \frac{x}{1+9+6x+x^2}\,dx$$

$$= \int \frac{x}{1+(x+3)^2}\,dx$$

Let $x+3=u$

$dx = du$

$$= \int \frac{u-3}{1+u^2}\,du$$

$$= \int \frac{u}{1+u^2}\,du - 3\int \frac{1}{1+u^2}\,du$$

$$= \frac{1}{2}\ln(1+u^2) - 3\tan^{-1}(u) + C$$

$$= \frac{1}{2}\ln\left(1+(x+3)^2\right) - 3\tan^{-1}(x+3) + C$$

$$= \frac{1}{2}\ln\left(10+6x+x^2\right) - 3\tan^{-1}(x+3) + C$$

(16) 8. (a) Show that Show that $\int \sec^n x dx = \frac{1}{n-1} \tan x \sec^{n-2} x + \frac{n-2}{n-1} \int \sec^{n-2} x dx$ $(n \neq 1)$

(b) Use part (a) to evaluate the integral $\int \sec^5 x\, dx$

(a) $I = \int \sec^n x\, dx = \int \sec^{n-2} x \sec^2 x\, dx$

$= \sec^{n-2} x \int \sec^2 x\, dx - \int \left[(\sec^{n-2} x)' \int \sec^2 x\, dx \right] d$

$= \sec^{n-2} x \tan x - (n-2) \int \sec^{n-3} x \sec x \tan^2 x$ d

$= \sec^{n-2} x \tan x - (n-2) \int \sec^{n-2} x \tan^2 x\, dx$

$= \sec^{n-2} x \tan x - (n-2) \int \sec^{n-2} x (\sec^2 x - 1)$ d

$= \sec^{n-2} x \tan x - (n-2) \int \sec^n x\, dx + \int (n-2) \sec^{n-2}$

$= \sec^{n-2} x \tan x - (n-2) I + (n-2) \int \sec^{n-2} x\, dx$

$I + (n-2) I = \sec^{n-2} x \tan x + (n-2) \int \sec^{n-2} x\, dx$

$(n-1) I = \sec^{n-2} x \tan x + (n-2) \int \sec^{n-2} x\, dx$

$I = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x\, dx$

(b) $\int \sec^5 x\, dx = \frac{1}{4} \tan x \sec^3 x + \frac{3}{4} \int \sec^3 x\, dx$

$= \frac{1}{4} \tan x \sec^3 x + \frac{3}{4} \left[\frac{1}{2} \tan x \sec x + \frac{1}{2} \int \sec x\, dx \right]$

$= \frac{1}{4} \tan x \sec^3 x + \frac{3}{8} \tan x \sec x + \frac{3}{8} \ln|\sec x + \tan$

$+ C$

(8) 9. $\int \sec^{-1} x dx$

$$= \int 1 \cdot \sec^{-1} x \, dx$$

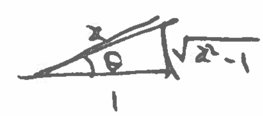
$$= \sec^{-1} x \int 1 \, dx - \int \left[(\sec^{-1} x)' \int 1 \, dx \right] dx$$

$$= x \sec^{-1} x - \int \frac{1}{x\sqrt{x^2-1}} \cdot x \, dx$$

$$= x \sec^{-1} x - \int \frac{1}{\sqrt{x^2-1}} \, dx$$

Let $x = \sec\theta$

$dx = \sec\theta \tan\theta \, d\theta$



$$= x \sec^{-1} x - \int \frac{1}{\sqrt{\sec^2\theta - 1}} \sec\theta \tan\theta \, d\theta$$

$$= x \sec^{-1} x - \int \frac{1}{\tan\theta} \sec\theta \tan\theta \, d\theta$$

$$= x \sec^{-1} x - \int \sec\theta \, d\theta$$

$$= x \sec^{-1} x - \ln|\sec\theta + \tan\theta| + C$$

$$= x \sec^{-1} x - \ln\left|x + \sqrt{x^2-1}\right| + C$$