M. Sadeghi.

CARLETON UNIVERSITY

FINAL
EXAMINATION
August 2005

DURATION: 3 HOURS

No. of Students

Department Name & Course Number: Mathematics and Statistics MATH 1107
Course Instructor(s) Dr. Mohammad R. Sadeghi

AUTHORIZED MEMORANDA

Calculators are not allowed.

Students **MUST** count the number of pages in this examination question paper **before** beginning to write, and report any discrepancy immediately to a proctor. This question paper has 12 pages.

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Family Name : ____________ **First (Given) Name :** ____________

Student Number : ____________ **Section :** ____________

Instructor: ____________

Problem	Maximum Mark	Actual Mark
1	8	
2	4	
3	4	
4	8	
5	4	
6	8	
7	4	
8	6	
9	4	
10	10	
Total	60	

M, Sadegh

1- In the following, circle your answer. Note that right answer has 0.5 mark. wrong answer **-0.5** mark and no answer has 0 mark. The total minimum mark of this question is zero.

T F - Row Echelon form of a matrix is not unique and reduced row echelon form is unique.
T F - The equation $\mathbf{Ax} = \mathbf{b}$ is consistent if its augmented matrix has a pivot position in every row.
T F - The columns of an $m \times n$ matrix A are linearly dependent if and only if the equation $\mathbf{Ax} = \mathbf{0}$ has the trivial solution.
T F - A linear transformation from R^n to R^m is onto if and only if each row of its standard matrix has a pivot position.
T F - If $v_1, v_2, \ldots, v_p$ are in R^n, then Span$\{v_1, v_2, \ldots, v_p\}$ is the same as the column space of the matrix $[v_1 v_2 \cdots v_p]$.
T F - The column space of a matrix A is the set of solutions of $A\mathbf{x} = \mathbf{b}$.
T F - The dimensions of Col A and Nul A add up to the number of rows of A
T F - The dimension of Nul A is the number of free variables in the equation $A\mathbf{x} = 0$.
T F - If the columns of A are linearly independent, then $\det A = 0$.
T F - $\det A^T = (-1) \det A$.
T F - If A is invertible then $(\det A)(\det A^{-1}) = 1$.
T F - A row replacement operation on A does not change the eigenvalues.
T F - If $\lambda = 0$ is an eigenvalue of A, then A is not invertible.
T F - A matrix with orthonormal columns is an orthogonal matrix.
T F - An orthogonal matrix is invertible.
T F - For each y and each subspace W, the vector $y - proj_W y$ is orthogonal to W.

M, Sadegh

2- The following system is given:

$$\begin{aligned} x + y + 3z &= 1 \\ 3x + 5y + 10z &= 6 \\ 2x + 4y + (9+h)z &= 5 \end{aligned}$$

a- For what value(s) of h the system has infinitely many solutions?
b- For what value(s) of h the system is inconsistent?
c- For what value(s) of h the system has a unique solution?

M. Soly

3- Let

$$\mathbf{v_1} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix}, \quad \mathbf{v_3} = \begin{bmatrix} -4 \\ 3 \\ -1 \end{bmatrix}, \mathbf{v_4} = \begin{bmatrix} 3 \\ 9 \\ 13 \end{bmatrix}.$$

Does the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ linear independent? Justify your answer.

4- Let $T : R^n \to R^m$ is defined by:

$$T\left(\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}\right) = \begin{bmatrix} x+y+z \\ 2y+2z \\ x+y+z+w \end{bmatrix}$$

a- What is n and m? Find $T(1,2,3,4)$.
b- Find the standard matrix of T.
d- Is T one-to-one? Justify your answer.
e- Is T onto? Justify your answer.

M. Sadr

5- Let $A = \begin{bmatrix} -2 & -7 & -9 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{bmatrix}$. Find the **third** column of A^{-1}.

M. Saleh

6- The matrix $A = \begin{bmatrix} 1 & 2 & -4 & 3 & 3 \\ 5 & 10 & -9 & -7 & 8 \\ 4 & 8 & -9 & -2 & 7 \\ -2 & -4 & 5 & 0 & -6 \end{bmatrix}$ is row equivalent to $B = \begin{bmatrix} 1 & 2 & -4 & 3 & 3 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

a- Find a basis for Col$A$.

b- Find a basis for NUL A.

c- What is the rank of A?

d- Verify the rank theorem.

M. Sa[illegible]

7- Let

$$\mathbf{b_1} = \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}, \quad \mathbf{b_2} = \begin{bmatrix} 7 \\ 5 \\ -6 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 11 \\ 0 \\ 7 \end{bmatrix}$$

Suppose B $= \{\mathbf{b_1}, \mathbf{b_2}\}$ is a basis for a subspace of . Find $[X]_{\mathrm{B}}$, coordinate vector of X relative to B.

M, Saad

8- Let $A = \begin{bmatrix} 0 & -4 & -6 \\ -1 & 0 & -3 \\ 1 & 2 & 5 \end{bmatrix}$.

a- Find all the eigenvalues of A and a basis for each corresponding eigenspaces.

b- Is A diagonalizable? If yes, find an invertible matrix P and a matrix D such that $A = PDP^{-1}$.

M. Saah

9- Let $A = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$. Find all the eigenvalues of A. If possible, diagonalize A.

10- Let

$$\mathbf{u_1} = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{u_2} = \begin{bmatrix} 5 \\ -2 \\ 4 \\ -2 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ -1 \end{bmatrix},$$

and $W = span\{\mathbf{u_1}, \mathbf{u_2}\}$.
a- Show that $\{\mathbf{u_1}, \mathbf{u_2}\}$ is an orthogonal set.
b- Find the orthogonal projection of x onto W.
c- Find the distance from x to W.
d- What is the nearest point in W to x?
e- Find an orthonormal basis for W.
f- Find a basis for the orthogonal complement of W, i.e., a basis for $W^{\perp}$.

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