University of Ottawa Faculty of Administration

ADM 2303: STATISTICS FOR MANAGEMENT I

FINAL EXAMINATION December 17, 2001

NAME S.N. Section: A B C D

Time: 3 hours Total marks:65 + bonus 2 *Put your name on THIS sheet – YOUR EXAM IS UNIQUE.*

ALL ANSWERS (INCLUDING BRIEF EXPLANATIONS) GO ON THE ANSWER SHEET. THE EXAM QUESTION SHEETS WILL **NOT** BE MARKED, though space is provided here for your rough work. The question sheets **must** be deposited in the box provided. NOTE THAT THERE ARE MARKS FOR EXPLAINING YOUR ANSWERS, SO MAKE SURE YOU INCLUDE BRIEF EXPLANATIONS ON THE **ANSWER** SHEET. THERE ARE MARKS FOR IDENTIFYING PROBABILITY DISTRIBUTIONS. Calculators, 1 sheet of notes, on 8.5" by 11" paper (no stick-ons!). You do **not** need to interpolate, but take the nearest table value.

1. An epidemiologist notes that only 8% of those using medicated mosquito nets in an endemic area get malaria, while 65% of those not using nets become ill. The epidemiologist notes that in the group he studied, people only used the medicated nets or no net at all, and that about 32% of people became ill with malaria. In his records, the sheet of paper giving the proportion using nets is unfortunately discarded.

a) [4] What is the proportion of people using the nets?

b) [2] If someone has become ill, what is the probability they were using a net?

2. The Polluchem factory has been discharging imbecillium, a chemical pollutant that can render people incapable of solving statistics problems. The amounts discharged per day have historically been approximately Gaussian distributed with a mean per day of 2.8 g and a standard deviation of 0.46 g. The company states "our discharges are less than 3 g per day".

a) [3] What percentage of days will discharges be less than 2 g?

b) [2] Two days are chosen randomly. What is the probability that both will have discharges above 2 g?

c) [6] An Environment Ministry worker thinks Polluchem may be producing more imbecillium than it claims. A sample of 8 randomly chosen days show discharges as follows:

2.73 3.36 2.81 3.19 3.8 2.58 2.92 2.99

Given this sample of discharges, should the Environment worker conclude that discharges are now greater than the historical value of 2.8g suggested at the start of this question? Support your argument with an appropriate calculation, possibly approximate. Note that a historical standard deviation is available.

d) [5] If the Environment worker does NOT have a historical value for the standard deviation of discharges, should he/she conclude that discharges are now greater than the historical value? Support your argument with an appropriate calculation, possibly approximate.

e) [4] Peace Green decides to do his own monitoring of Polluchem. He computes that from 64 days, the mean discharge is 2.9 g with standard deviation 0.45 g. Should Peace conclude the discharges are greater than the historical value of 2.8g? Explain.

f) [1] In making his decision, does Peace need the discharge amounts to be Gaussian distributed.

g) [6] Peace knows that the newspaper editors do not understand statistics (even if they are not exposed to imbecillium). Peace therefore would like to make a statement like “on X out of Y days, Polluchem exceeds their own stated discharge limit of 3 g per day”. If the mean and standard deviation discharge are equal to that in Peace Green’s sample, what is the probability that 15 or more of Peace’s 64 sample days will exceed a 3 g discharge?

Q 3. BatteryCo makes store-brand discount AA alkaline cells. BuyBatt, an electronic store, requires that the cells must have a voltage on delivery between 1.40 and 1.43 volts. (There are other specifications, but they will not be considered here.) BatteryCo develops a manufacturing process that they believe gives a mean voltage of 1.416 v with standard deviation of 0.03 v.

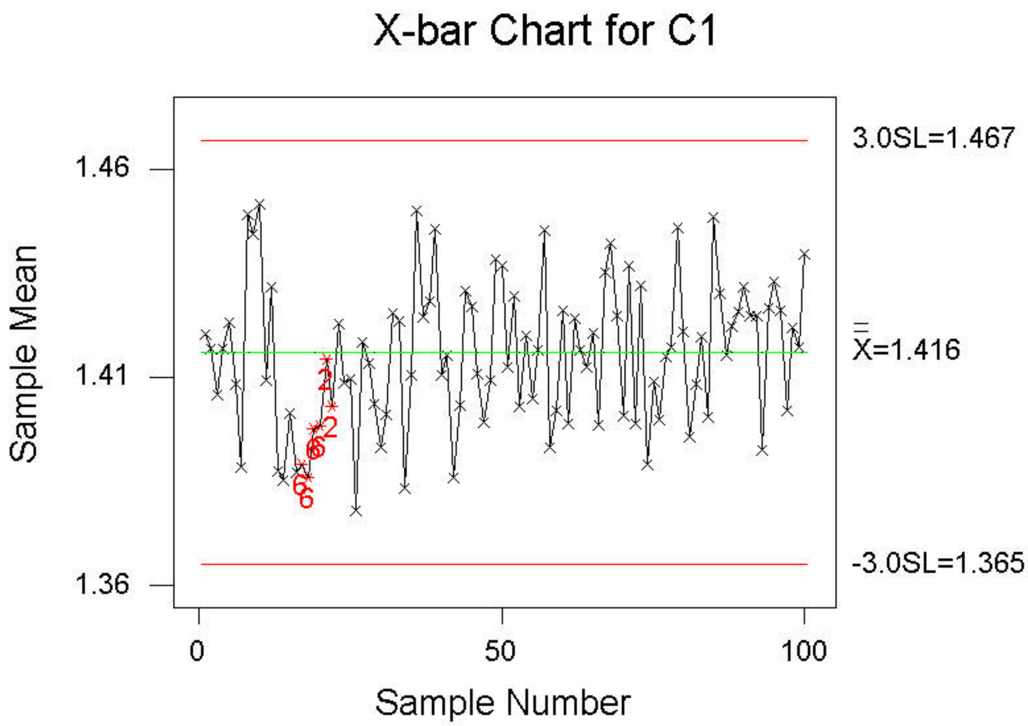
Overleaf you will find some quality control charts for the battery voltages. Additional output from Minitab you ***may*** find useful follows:

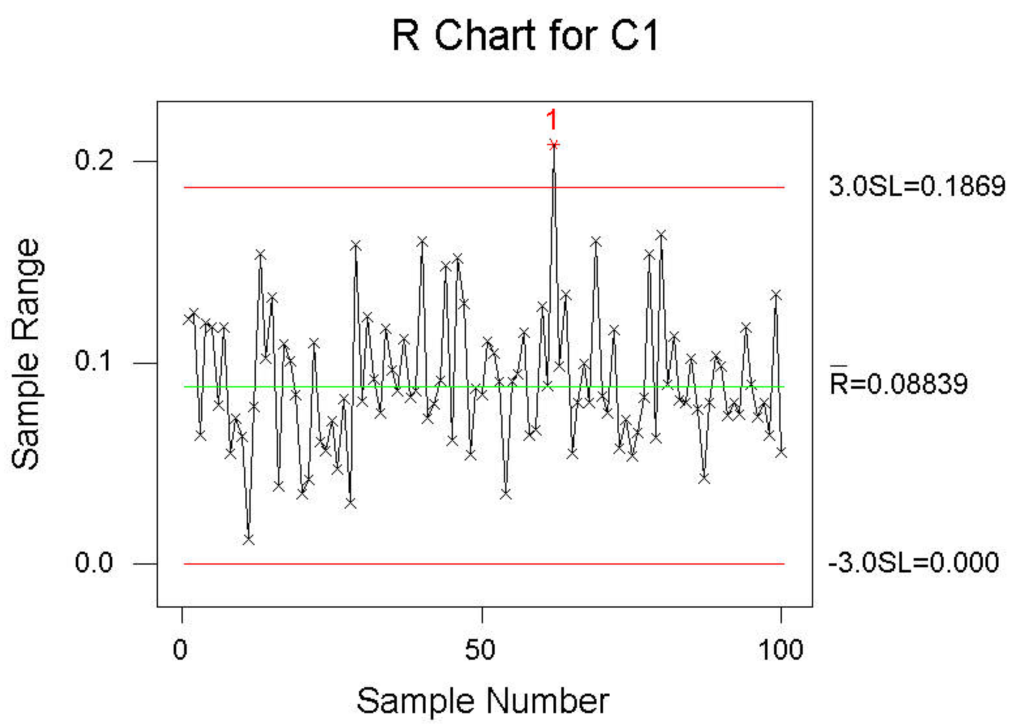
```
MTB > %Rxbarr ;
SUBC>   Csub C1 10;
SUBC>   Mu 1.416;
SUBC>   Sigma .038;
SUBC>   Rbar;
SUBC>   Test 1 2 3 4 5 6 7 8.
Test Results for Xbar Chart
TEST 2. 9 points in a row on same side of center line.
Test Failed at points: 15 16 17

TEST 5. 2 out of 3 points more than 2 sigmas from center line
        (on one side of CL).
Test Failed at points: 9

TEST 6. 4 out of 5 points more than 1 sigma from center line
        (on one side of CL).
Test Failed at points: 10

Test Results for R Chart
TEST 1. One point more than 3.00 sigmas from center line.
Test Failed at points: 31
```


X-bar Chart for C1
3.0SL=1.467
1.46
Sample Mean
1.41
X=1.416
1.36
-3.0SL=1.365
0
50
100
Sample Number


R Chart for C1
1
0.2
3.0SL=0.1869
Sample Range
0.1
R=0.08839
0.0
-3.0SL=0.000
0
50
100
Sample Number

a) [2] What can you say about the process of making these batteries from the R-chart?

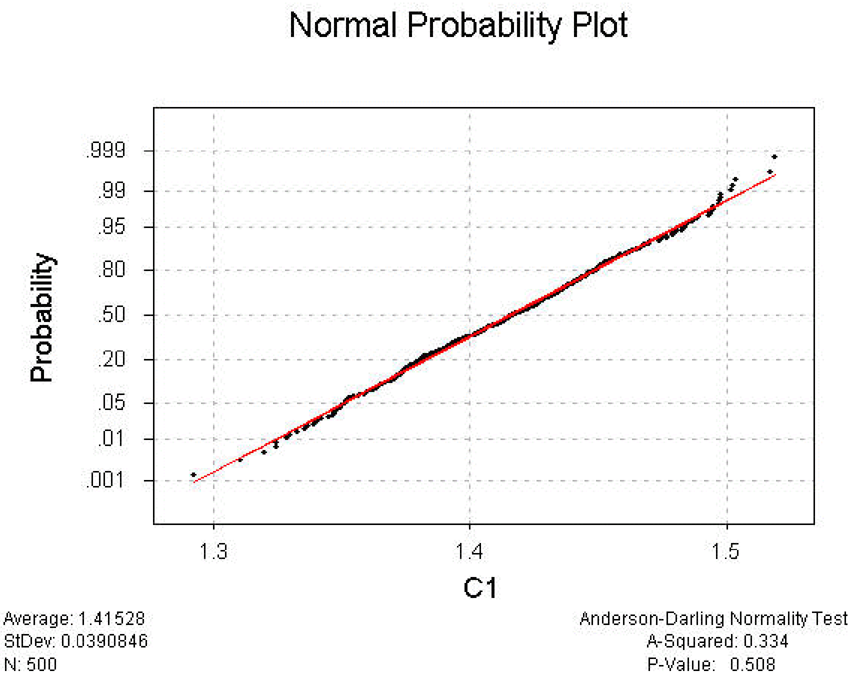
b) [2] What can you say about the process of making these batteries from the Xbar-chart?

c) [2] Should you accept batteries from the process ***as it is performing at the end of the data series***?

You are suspicious about the distributional properties of your battery voltages data, so draw the following two graphs:

```
          Histogram of C1   N = 500
Each * represents 5 observation(s)

Midpoint         Count
    1.30             1  *
    1.32             6  **
    1.34            18  ****
    1.36            33  *******
    1.38            79  ****************
    1.40            80  ****************
    1.42           103  *********************
    1.44            86  ******************
    1.46            52  ***********
    1.48            27  ******
    1.50            13  ***
```



d) [2] From these graphs, can you conclude the data is Gaussian distributed? Explain briefly.

e) [2] Joe buys some batteries from a street hawker at a highly discounted price. When he takes them to the lab and measures the voltages, he gets data giving the histogram at the right. Can you suggest what could give rise to this distribution of voltages. (We are looking for an answer which uses statistical thinking!)

```
          Histogram of C1   N = 500
Each * represents 5 observation(s)

Midpoint         Count
    1.30             1  *
    1.32             6  **
    1.34            18  ****
    1.36            33  *******
    1.38            79  ****************
    1.40            80  ****************
```

Q 4. SuperComm's new communication satellite suffers an average of 1.8 cosmic ray "hits" per year, and these may occur at any time.

a) [4] SuperComm will launch a shielded backup satellite in 2 years. What is the probability there will be no cosmic ray hits during that time?

b) [4] What is the probability there will be less than 3 hits in the 2 years?

c) [6+2] Each cosmic ray hit has a the probability of 10% that it will knock out the satellite. Show that the probability that the satellite will survive until the backup is launched is ***at least*** 0.6? HINT: Deal with each possible number of hits separately, e.g., no hits, 1 hit, 2 hits, ... There is a bonus 2 marks for the calculation of the probability of survival until the backup is launched. It is useful to know the Taylor-McLaurin expansion of exp(x) is

$$\exp(x) = \sum_{k=1}^{\infty} x^k / k!$$

d) [3] What is the probability of no cosmic ray hits on the satellite in the next 6 months?

Q 5. A large hospital needs 7 operating theatre technicians to fully staff the theatre out of a total of 9 on staff in one of its theatre teams. It happens that two workers are unavailable due to sickness or vacation. As the clerk of the Emergency Room who is phoning staff, you do not know about the sickness or vacation, but you phone the team members in a random order.

a) [3] What is the probability that the ***second*** person called will be one of those that is unavailable?

b) [5] What is the probability of finding both unavailable workers in the first four calls?

c) [2] Noting that there are 9 workers on the team (we will assume they are interchangeable), and two are unavailable, what is the probability you need make only 7 calls to alert the team?

NAME: S.N. Section: A B C D E

1 a) [5]

b) [3]

2 a) [4]

b) [2]

c) [6]

d) [5]

e) [4]

f) [1]

g) [6]

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3 a) [2]

b) [2]

c) [2]

d) [2]

e) [2]

4 a) [4]

b) [4]

c) [6]

+

d) [3]

5 a) [4]

b) [5]

c) [3]

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NAME:Solutions S.N. Section: A B C D E

1 a) [5]
1 setup P(malaria | net) = 0.08
1 exp or diag P(malaria | nonet) = .65
P(malaria) = .32
2 P(malaria) = P(net and malaria) + P(nonet and malaria) = .32
= P(malaria | net)*P(net) + P(malaria | nonet) * (1 - P(net))
= .08 * P(net) + .65 - .65 *P(net)
1 .57 *P(net) = .65 -.32, so P(net) =0.578947368421053
or nearly 58% are using nets

b) [3]
1 setup P(net | malaria)
= P(net and malaria)/P(malaria)
1 work =P(malaria | net) * P(net) / .32
1 answer = 0.1447368
or 14.5% approx.

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2 a) [4]
use mu= 2.8 sigma= 0.46 (historical values)
P(X < 2g | mu = 2.8, sigma=0.46) = P(z < (2-2.8)/.46) =
P(z <-1.74) = +0.5-0.4591 = 0.041
1 setup, 1 z, 1 lookup, 1 conclusion

b) [2]
P(both above 2 g)
= (1-0.041)^2 = 0.919681
1 setu[p, 1 answer

---

c) [6]

X_bar= 3.0475 s = 0.39271 (1 each, may be in (d))
1 Use Gaussian distribution (you are given this info)
If mu = 2.8 g/day then
2 P(X_bar > ~~2.9225~~ (3.0475) g/day)= P(z > 1.52)= 0.0643
1 Prob > 5% so cannot conclude that outflow has increased

d) [5]

1 Now have no historical value, so must use t stat.
(Popn still Gaussian)
1 t = (X_bar - mu)/(s/sqrt(n))
1 P(t > 1.7826)= ?
But with 7 d of f (1 mark)

0.1	0.05	alpha
1.4149	1.8946	t7(alpha)

so probability is between 5% and 10%
Still cannot say outflow has increased. (1)

---

e) [4]

n "large" so CLT applies (1 mark)
P(Xbar >= 2.9 | mu=2.8, sigma=0.46 or 0.45) = P(z>1.74) = 0.0409
2 marks
Because prob is small (< 5%) conclude there has been increase
(1 mark) (argument important rather than conclusion)

f) [1]

No! CLT (must be mentioned)

---

g) [6]

2 Assume chance discharge > limit is P(>3g | 1 day)= P(z> 0.22)
= 0.4129 =p
1 P(K >= Klim | n, p) ~= Klim= ~~20~~ (15) 64 =n
1 P(z > (Klim-0.5 - np)/sqrt(npq))= np= 26.4256 npq= 15.51446976
1 cont corr P(z > -1.76)= 0.9608 1mark both mu, sigma

(Changed exam but not sol'n)

(These will also change)

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3 a) [2]
1 The R chart has just 1 point (#31) where it exceeds the 3-sigma bound.
1 While we could say that the process is uncontrollable, given that this point is now passed, we would likely accept that the process is now controllable.
1 mark for noting the excursion, 1 for word "controllable" or equivalent (NOT "in control" however).

b) [2]
There are no excursions across the control lines, but early in the set of samples, we see some points that fail some of the X_bar tests. Likely would accept that the process is now "in control" (THOSE WORDS).
1 mark for noting tests, 1 for decision (either way OK, but with justification).

c) [2]
Yes.
At the end of the series, the process is both controllable and in control.
MUST justify for both marks

d) [2]
Data gives mound shape to histogram and more or less straight NPP
Mention both for full marks

e) [2]
Batteries with good voltage have been cherry-picked and only low voltage ones sold by hawker.
Must give clear explanation like this for full marks.

4 a) [4]
1 Poisson. Rate is 1.8 hits/year(May occur any time.)
1 In 2 years, expect 3.6 hits
P(k)=mu^k * exp(-mu)/k!
1 P(0)= exp(-3.6)= 0.0273237
1 About 2.73 % chance of no hits in 2 years.

b) [4]
1 P(k<3) = P(0) + P(1)+P(2) =
2 =exp(-3.6)*(1+3.6+3.6^2/2) = 0.302747
1 About 30.2%

c) [6]

3 marks for showing diagram or statement that we need to compute prob of k hits, then work out prob of surviving those hits

2 marks for setting up P(k hits) * (0.9^k) = Prob survive k
1 for showing sum > 0.6 (at k = 5)
+2 for showing infinite sum using Taylor series is 0.697676

d) [3]

In 6 months expect 0.9 hits (1)
P(0)= 0.40657 or 40% chance of no hits
(2 for finishing)

5 a) [4]
1 P(#2 not available) = P(1 OK, 2 not) + P(1 not, 2 not)
1 = 7/9 * 2/8 + 2/9 * 1/8 =
1 = (14 + 2) / 72 = 0.222222222222222

b) [5]
2 Hypergeometric: N = 9, S = 2, n=4, k=2
2 P(k | N, S, n) = C(N-S, n-k) * C(S, k)/C(N, n)
= 21 1 / 126
1 = 0.166666666666667

c) [3]

2 Hypergeometric N=9, S=2, k=0, n=7
1 = 0.0277777777777778