University of Ottawa Faculty of Administration

ADM 2303: STATISTICS FOR MANAGEMENT I

FINAL EXAMINATION December 11, 2000 1900-2200

NAME: S.N. Section: A B C D

Time: 3 hours Total marks:65 + bonus 2 *Put your name on THIS sheet in case you make a transfer error.*

ALL ANSWERS (INCLUDING BRIEF EXPLANATIONS) GO ON THE ANSWER SHEET. THE EXAM QUESTION SHEETS WILL **NOT** BE MARKED, though space is provided here for your rough work. The question sheets **must** be deposited in the box provided. NOTE THAT THERE ARE MARKS FOR EXPLAINING YOUR ANSWERS, SO MAKE SURE YOU INCLUDE BRIEF EXPLANATIONS ON THE **ANSWER** SHEET. THERE ARE MARKS FOR IDENTIFYING PROBABILITY DISTRIBUTIONS. Calculators, 1 sheet of notes, on 8.5" by 11" paper (no stick-ons!). You do **not** need to interpolate, but take the nearest table value.

Q 1. Thread Mills produces bolts of cloth that average 2.2 defects per square metre.

a) [4] What is the probability there will be no defects in 2 square metres;

b) [4] There will be more than 2 defects in 2 square metres.

c) [4] Thread Mills sends out sample books that use squares of cloth that are 15 cm on a side. What is the probability a single such square has no defects?

d) [5] Regardless of your answer in (c), use a probability of 0.97 of no defect. If a dozen sample books are prepared, what is the probability that two of them have defects?

Q 2. A series of eight electrical parts are connected in such a way that if one part fails, the system will not operate. It happens that two parts have failed. You will inspect the parts in a random order.

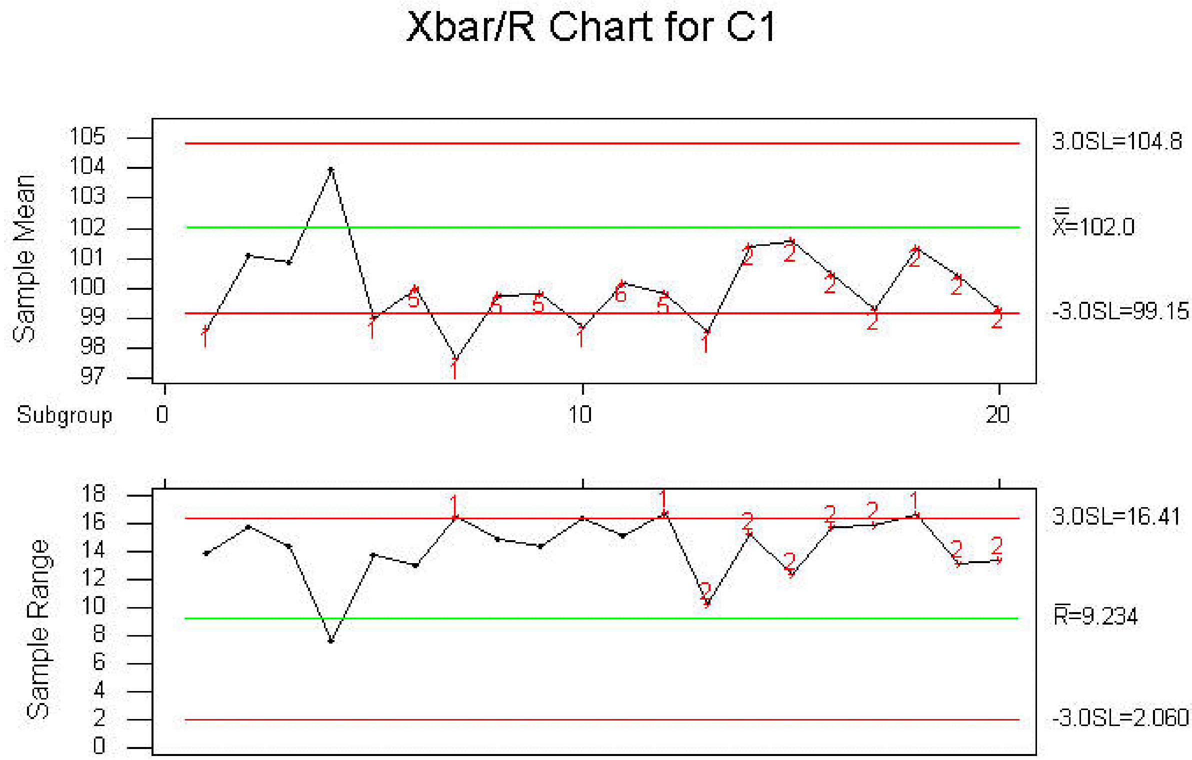
a) [2] What is the probability that the first part that is inspected will be one of those that has failed?

b) [4] What is the probability of finding both failed parts if four parts are inspected?

c) [2] The management wants at least 70% probability that BOTH bad parts have been found. If two parts out of 8 have actually failed, how would you calculate how many parts must be inspected to achieve this 70% probability of finding both bad parts. *NOTE: the calculation can be messy unless you are well organized. You do NOT need to do the calculations, only describe how they would be done.*

Bonus mark [2]: BRIEFLY show the calculation results.

Q 3. The Goodman Tire and Rubber Company periodically tests its tires for tread wear under simulated road conditions. To study and control its manufacturing processes, the company uses control charts for both the average wear and the range of the wear. The design for the particular tires aims to have wear under the simulated conditions between 0.9 and 1.14 millimeters. 20 samples, each containing 10 radial tires, were chosen from different shifts. The results (in hundredths of millimeters i.e., 0.01 mm units) are shown in the graphs below.



Additional output from Minitab you ***may*** find useful follows:

```
Test Results for Xbar Chart
TEST 1. One point more than 3.00 sigmas from center line.
Test Failed at points: 1 5 7 10 13

TEST 2. 9 points in a row on same side of center line.
Test Failed at points: 13 14 15 16 17 18 19 20

TEST 5. 2 out of 3 points more than 2 sigmas from center line (on one side of CL).
Test Failed at points: 6 7 8 9 10 12 13

TEST 6. 4 out of 5 points more than 1 sigma from center line (on one side of CL).
Test Failed at points: 7 8 9 10 11 12 13 20

TEST 8. 8 points in a row more than 1 sigma from center line (above and below CL).
Test Failed at points: 10 11 12 13

Test Results for R Chart
TEST 1. One point more than 3.00 sigmas from center line.
Test Failed at points: 7 12 18

TEST 2. 9 points in a row on same side of center line.
Test Failed at points: 13 14 15 16 17 18 19 20
```

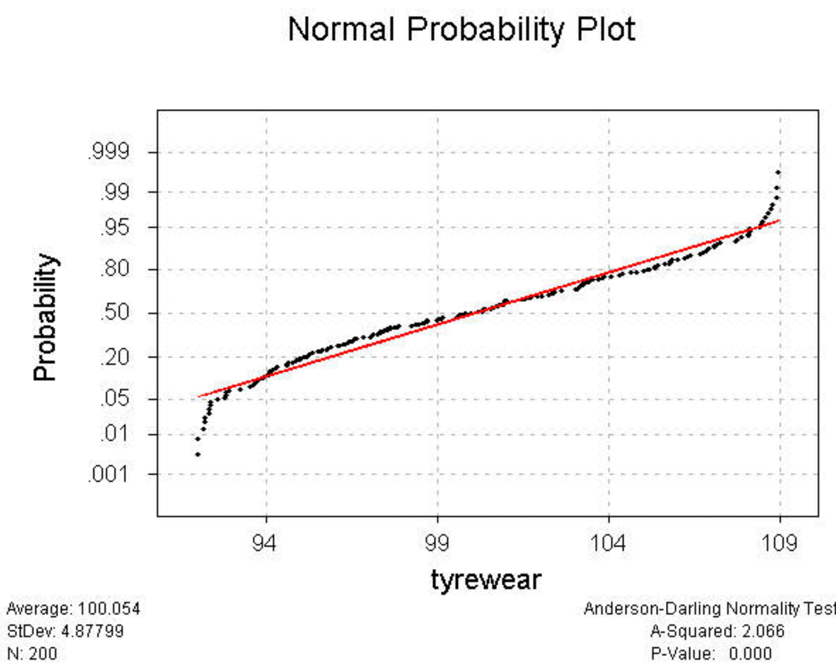
a) [2] What can you say about the process of making these tires from the R-chart?

b) [2] What can you say about the process of making these tires from the Xbar-chart?

You are suspicious about the distributional properties of your tire wear data, so draw the following two graphs:

```
Histogram of tirewear   N = 200

Midpoint        Count
      92           14  **************
      94           25  *************************
      96           25  *************************
      98           26  **************************
     100           29  *****************************
     102           16  ****************
     104           23  ***********************
     106           21  *********************
     108           21  *********************
```



c) [2] From these graphs, can you conclude the data is Gaussian distributed? Explain briefly.

d) [2] Based on the two graphs above, you decide to graph the quantile of the data versus the sorted data. (If X(i) is the sorted observation at rank i, the quantile q(i) = (i - 0.5)/n where n is the total number of observations. Thus the graph is very similar to a cumulative frequency graph or ogive.) This graph is at the right. Using it, and the graphs above, can you suggest what shape the tire wear data has? If you know the name of the distribution you can use that name, but a description of the shape is enough. Explain BRIEFLY.

4. Weights of fish caught by a trawler over the past few years have been approximately normal, with a mean of 4.5 lb and a standard deviation of 0.5 lb. (lb. = pounds Imperial, the British/American weight unit)

a) [3] What percentage of the fish weigh less than 4 lbs?

b) [2] If two fish are chosen randomly, what is the probability that one will weigh more than the mean and one less?

c) [6] A Fisheries and Oceans researcher thinks that foreign trawlers have been illegally overfishing the stock so that fish are no longer as big as previously. A sample of 8 randomly chosen fish are weighed:

4.17   3.36   4.81   3.89   4.80   4.38   2.92   3.60

Given this sample of fish, should the Fisheries researcher conclude that fish now being caught are smaller? Support your argument with an appropriate calculation, possibly approximate.

d) [5] If the Fisheries researcher does NOT have a historical value for the standard deviation of fish weights, should he/she conclude that fish now being caught are smaller? Support your argument with an appropriate calculation, possibly approximate.

e) [4] Joe Poisson decides to weigh his entire catch fish by fish. He computes that from 81 fish, the mean weight is 4.35 lbs. Should Joe conclude the fish are smaller than the historical size? Explain.

f) [1] In making his decision, does Joe need the fish weights to be Gaussian distributed.

g) [6] Joe sells his fish to Frank's Fish Food Fantasy restaurant. The restaurant wants all the fish to be a similar size, but only 65% of the fish in a catch are generally suitable. Joe would like to sell 60 fish to the restaurant out of his 81 fish catch. What is the probability 60 or more of the fish caught are suitable.

5. Police records reveal that 10% of accident victims who are wearing seat belts sustain serious injuries, while 50% of those who are not wearing seat belts sustain serious injury. Police estimate that 90% of the people riding in cars use seat belts. Police are called to investigate a two-car accident in which one person is seriously injured.

a) [4] Estimate the probability that the injured person was wearing his seat belt at the time of the crash.

b) [2] The driver of the second car was not seriously injured. Estimate the probability that he was wearing his seat belt.

NOTE– following pages shrunk from 8.5" by 14" to fit on 8.5" by 11" (avoids printer problems)

ADM 2303 - STATISTICS FOR MANAGEMENT I December 11, 2000 1900-2200

NAME: S.N. Section: A B C D

1 a) [4]

b) [4]

c) [4]

d) [4]

2. a) [2]

b) [4]

c) [2]

c bonus [2]

Q 3 a) [2]

b) [2]

Q 3c) [2] | d) [2]

Q 4. a) [3] | b) [2]

c) [6] | d) [5]

e) [4] | f) [1]

g) [6]

5 a) [4] | b) [2]

ADM 2303 - STATISTICS FOR MANAGEMENT I — December 11, 2000 1900-2200

NAME: S.N. Section: A B C D

1 a) [4]

Poisson mu = 2 m * 2.2 defects/m = 4.4 def.
P(K=0 | mu=4.4) = exp(-4.4) = 0.012277

1 mark Poisson
1 mark for working out mu
1 mark for formula
1 mark answer

b) [4]

P(K > 2 | mu=4.4) = 1 - P(0) - P(1) - P(2)
= 1 - exp(-4.4) * (1 + 4.4 + 4.4^2 / 2)
= 1 - 0.012277*(1 +4.4+9.68)
= 1-0.185142 = 0.814858

1 mark Poisson / setup with mu = 4.4
1 mark for 1 - ...
1 mark for working
1 mark for answer

---

c) [4]

Recompute mu from area = .15^2 m
= .0225 m^2
mu = 2.2 * .0225 defects = 0.0495
P(K = 0 | .0495) = exp(- 0.0495)
= 0.9517

d) [4]

Binomial n = 12, p = 0.03
P(K=2 | n=12, p=0.97)
= C(12, 2) 0.97^10 0.03^2
= 66 * 0.737424 * 0.0009
= 0.043803

1 binomial, 1 n, p
1 working, 1 answer

---

2. a) [2]

The situation is hypergeometric, but this part can be done directly.
P(first bad of 8 when 2 actually bad)
= 2/8 = 1/4 = 0.25
1 setup, 1 answer

b) [4]

Hypergeometric
P(K=2 | N=8, S=2, n=4)
= C(S,K)* C(N-S,n-K)/C(N,n)
= C(2,2)* C(6,2)/C(8,4)
= 1 * (6 * 5 / 2) / (8 * 7 * 6 * 5 / 4*3*2)
= 15/70 = .21429

1 mark hypergeometric
1 mark setup (parameters)
1 mark working, 1 answer

---

c) [2]

Need to find the sample size n such that
P(K=2 | N=8, S=2, n) > 0.7

Note that cannot have n < 2
so try n = 2, 3, 4, 5, ...
until P(K=2 | n) >0.7

1 mark for hypergeometric setup,
1 for explaining n = 2, 3, ...

c bonus [2]

P(2) = (2/8)(1/7) = 2/56 = .036
P(3) = 1*6/56
P(4) = 15/70
P(5) = 20/56
P(6) = 15/28
P(7) = 6/8 <-- ***

Q 3 a) [2]

The R-chart shows that the variability of the process is too great - thus the process in not CONTROLLABLE

1 mark variability, 1 NOT CONTROLLABLE

b) [2]

Though the process is not controllable, all the samples are within the specifications.
Note that students could also point out that

2 marks for explaining within specs.,
1 if only "in control"
Expect very few students to get this perfectly right.

30 this page (28 + 2 bonus)

Q 3c) [2]
NOT Gaussian – not “mound shaped” or
“Bell shaped” from histogram
Normal probability plot NOT on line

2 marks for explanation, but MUST
mention both graphs for 2 marks

d) [2]
The distribution is UNIFORM
Can see this from the histogram,
and also the cumulative probability
increases uniformly (linearly) with
the sorted observations.

2 marks for reasonable explanation

Q 4. a) [3]
P(W < 4) = P(z < (4 - 4.5) / .5)
= P(z < -1) = 1 - C(1) = 1 - .3413
= 0.1587

1 mark setup, 1 for z value and lookup
1 mark answer

b) [2]
P(fish W > 4.5) = 0.5 = P(W < 4.5)

P(1 fish > 4.5 lbs and 1 fish < 4.5 lbs)
= P(Fish A >) P(Fish B <) + P(Fish A <) P(fish B>)
= .5 * .5 + .5 * .5 = .5

1 explanation, 1 answer

---

c) [6]
Xbar = 3.99125 lbs, s = 0.677905
Because weights are Gaussian, the Xbar
is also Gaussian distributed (EXACTLY)
with mean 4.5 lbs and s.d. 0.5/sqrt(n)
n = 8 so sigma(Xbar)= 0.5/sqrt(8)=0.17678
P(Xbar <= 3.99125) =
= P(z < (3.99125 - 4.5)/.17678)
= P(z < - 2.8777) = 0.5 - 0.498 = 0.002

1 Xbar, 1 s, 1 Gaussian, 1 z, 1 lookup,
1 answer

d) [5]
Without sigma, use t statistic
P(Xbar < 4.5lbs) = P(t < (Xbar - 4.5)/(s/sqrt(8))
= P(t = -2.1227) using 7 degrees of freedom
P(t7 >= 1.895) = 0.05, P(t7 >= 2.365) =0.025
Therefore P(Xbar < 4.5 lbs) is between 2.5%
and 5%. Conclude fish unlikely to have pop’n
mean wt. of 4.5 lbs

(give 2 marks for Xbar and s in (c) if NOT done
there). 1 mark t, 1 mark t value, 1 mark for
table values, 1 for probs. and 1 mark conclusion

---

e) [4]
P(Xbar < 4.35lbs) = P(z < (4.35 - 4.5)/(.5/sqrt(81))
= P(z < -.3 * 9) = P(z < -2.7)
= 0.5 - 0.4965 = 0.0035

Yes. Fish are smaller than historically
since unlikely to get sample at random

f) [1]

It is not necessary to have
a Gaussian distribution of
fish weight as the sample
size (81) is “large” enough
that Xbar will be approx.
Gaussian by Central Limit
Theorem. [1 mark CLT]

---

g) [6]
Binomial situation [1 mark] n = 81, p = .65, [1 mark]
P(K >= 60 | n, p) = P(X >= 59.5 | mu = np = 52.65, sigma= sqrt(npq) = sqrt(18.42) = 4.293)
= P(z > (59.5 - 52.65)/4.293) = P(z > 1.596) (use 1.6) = 0.5 - .4452 = .0548 or about 5.5%
probability that Joe will have enough suitable fish.
1 mark correction for continuity, 1 mark z 1 mark table value, 1 mark answer

---

5 a) [4]

	Belt	No belt	
Injured	.09	.05	.14
NOT inj.			
	.9	.1	

P(I | B)=.1, P(I | not B)=.5
P(I & B)=.1 * .9 = .09
P(I & not B)= .1 * .5 = .05

P(B | I) = .09/.14 = .642
= P(B & I)/P(I)
Marks: 1 for definition of cond’l prob.
2 for setup/working, 1 for answer

b) [2]
(can give 2 working marks in (a) if NOT there)
P(B | not I) = .81 / .86 = 0.942
= P(B and not I)/P(not I)
(need table to do this)
1 for setup, 1 for answer

37 total page 2