MAT 1322 Test 1

Student Number: _____

•Time: 80 min.

• Only basic scientific calculators are permitted (non-graphing, non-programmable, no integration or differentiation capabilities). Notes or books are not permitted.

• Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.

• Write *only* in non-erasable ink (ball-point or pen), not in pencil. Cross out, if necessary, but do not erase or overwrite.

• The problems require complete and clearly presented solutions and carry part marks if there is substantial correct work towards the solution.

1.[4 points] (a) Consider the integral $\displaystyle\int_2^\infty \frac{1}{e^{2x}}\,dx$. Does it converge or diverge? If it converges, give its value.

(b) Use the Comparison Test to determine if the integral $\displaystyle\int_0^1 \frac{2+\cos(x)}{3x^3+\sqrt{x}}\,dx$ converges or diverges. If it converges, give an upper bound for its value.

2.[4 points] Use the method of cylindrical shells to find the volume of the solid obtained when the region bounded by $y = x^2$, $y = 1/x$, $x = 1$ and $x = 2$ is rotated around the y-axis. Include a sketch of the region and a typical cylinder (with dimensions) in your solution.

3.[4 points] A cylindrical tank of radius 2 m and height 4 m is full of water. Find the work required to pump all of the water to a point that is 2 m above the top of the tank. The density of water is $\rho = 1000$ kg/m^3 and the acceleration of gravity is $g = 9.8$ m/s^2. Define clearly all the variables that enter into your solution and provide a drawing which shows their meaning.

4.[3 points] Give the integral for the arclength of the curve $y = 2x^{3/2} + 1$ between $x = 0$ and $x = 9$ and then evaluate it.

5.[3 points] Give the integral for the average value of the function $f(x) = \ln(x)$ on the interval $[1, 3]$ and then evaluate it.

6.[2 points] Solve the initial value problem: $\sqrt{1-x^2}\,\dfrac{dy}{dx} = y+1\,, \quad y(0)=1\,.$