COMM 220

Analysis of Markets

Lecture Notes

Second Edition

HANY FAHMY

E. info@hftutoring.com
w. www.hftutoring.com
T. 514 979 4232

COURSE PACKAGE INFORMATION

This is a free copy of the first part of **COMM 220 course package**. If you are interested to know more about this course package or any other course you are currently taking, please take a minute and check our website

www.hftutoring.com

or call us on

514 963 3707 or 514 979 4232

You can also benefit from our WALK IN **promotions on all our services:**

- Weekly Tutorials
- Crash Courses
- Private Tutoring
- Free Advising with our instructors

Just visit our office located on 2015 Drummond (with Maisonneuve), 8th floor, Suite 822 **to learn more about these promotions.**

Looking forward to hearing from you and wishing you a great semester!

The Administration

HF Tutoring

Lecture Notes on The Price Theory

Winter, 2010

Hany Fahmy[1]

The basics of Supply and Demand

A. The Market Demand Curve

Definition 1 *the demand for a good or a service is the amount of that good or service that the consumers are willing and able to buy at a certain price in a given period of time. The market demand (aggregate demand) shows the total demand of all consumers in the market in a given period of time*

The demand curve expresses the relation between the price of a normal good and the quantity demanded of that good. Each point on the demand curve represents the maximum amount that the consumers are willing to pay at every price. The more the quantity consumed, the lower is the amount that the consumers are willing to pay. This follows the law of *diminishing marginal utility* and hence, the demand curve is negatively sloped. The negative relation between the quantity demanded and the price is referred to as the demand law

A.1. The Demand Law

$$Q_d = f(P)$$

The demand law states that the quantity demanded of a good is a function of its own price such that as the price of a good increases, the quantity demanded of that good must decrease.

This negative relation can be expressed from the following linear demand equation

$$Q_d = a - bP$$

where a is the intercept of the equation. It shows the maximum amount demanded by the consumers when the good is freely available, i.e., when $P = 0$. The parameter b is the slope of the demand equation as

$$Slope = \frac{\Delta Q}{\Delta P} = -b,$$

where the negative sign confirms the demand law stated above

A.2. The Determinants of Demand

$$Q_x^d = f(\underbrace{\underset{-ve}{P_x}}_{\text{change in } Q_d \text{ Movement}}, \underbrace{\underset{+ve}{P_S}, \underset{-ve}{P_C}, \underset{+ve}{Y}, \underset{+ve}{P^e}, \underset{-/+ve}{Taste}, \underset{+ve}{N}, \underset{-ve}{T}}_{\text{Shift in the demand curve up or down}}),$$

where P_S is the price of substitutes, P_C is the price of complementary goods, Y is the consumers' income, P^e is the expected price, N is the number of consumers, and T denotes taxes. The expected signs are shown beneath the variables

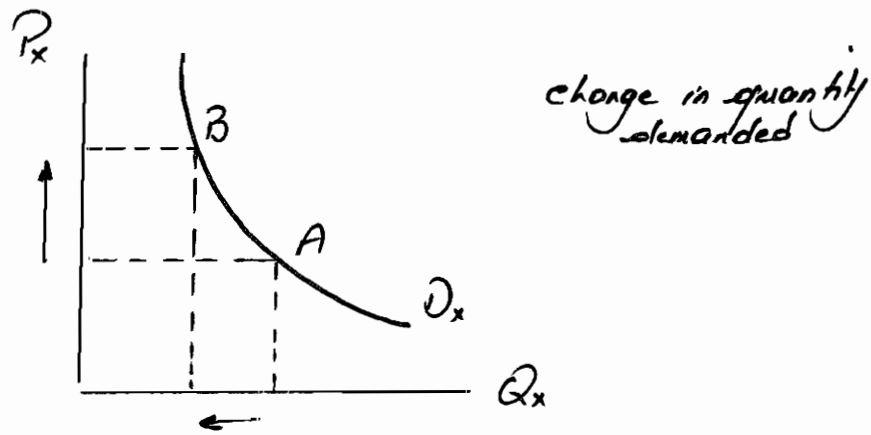
[1] hany.fahmy@hf-consultingco.com

Remark 2 *Before explaining the determinants of the demand, it is useful to distinguish between two concepts, the change in quantity demanded and the change in demand. A change in the quantity demanded is a change in the amount of a good demanded resulting solely from a change in price. Hence, Changes in quantity demanded are shown by movements along the demand curve. A change in demand, on the other hand, is a change in the amount of a good demanded resulting from a change in something other than the price of the good. It is represented by a shift (either upward or downward) of the demand curve.*

Now we consider each factor affecting the demand in turn as follows:

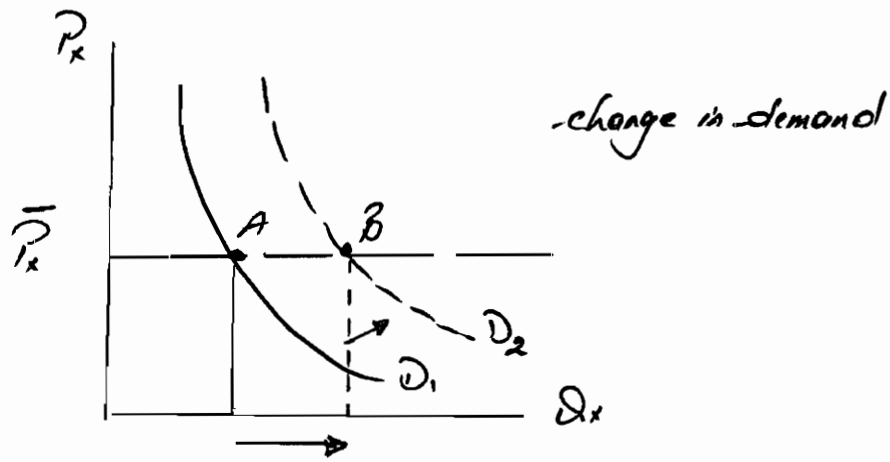
1. Change in the price of the good: $\triangle P_x$

   Given that x is a normal good; as P_x increases, holding all other factors constant, the quantity demanded of good x decreases and vice versa (the law of demand). This is represented by a movement along the demand curve



2. Change in the price of substitutes: $\triangle P_S$

   Two goods are said to be substitutes, if the consumer can substitute one for another and still maintain the same satisfaction. Consider, for instance, frozen yogurt and ice cream. If the price of substitute goods, the frozen yogurt, increases, the quantity demanded of ice cream would increase. This is represented by a right shift of the demand curve for ice cream (graph is omitted)



3 Change in the price of complements: $\triangle P_C$

Two goods are said to be complements, if they are consumed together, sugar and tea is a typical example. Consider a fall in the price of sugar, holding all other factors constant, the quantity demanded of the tea, the complementary good, increases. This is represented by a right shift of the demand curve for tea.

4 Change in income: $\triangle Y$

For normal goods, an increase in income leads to an increase in demand for that good. This is represented by a rightward shift of the demand curve.

5 Change in consumers' price expectations: $\triangle P^e$

If the consumers, for instance, anticipate that there will be a future price increase (inflation), then demand for the current products, with low prices, will increase. This is represented by a rightward shift of the demand curve.

6 Change in fashion and tastes

Changes in fashion and taste, e.g. food, clothing and entertainments, affect also the demand for a given good and causes the demand curve to shift either to the right or to the left.

7. Change in the number of buyers served by the market: $\triangle N$

An increase in the number of buyers, holding other factors constant, will shift the demand curve to the right and vice versa.

8. Change in government taxation policy: $\triangle T$

Whether the government increases or decreases the income tax, this would definitely affect the people's disposable income and consequently their demand. The higher the taxation, the lower the disposable income and the lower the demand in general.

B. The Market Supply Curve

Definition 3 *It refers to the quantity of a good or a service that suppliers are able and willing to offer for sale to the market at various market prices during a specified period of time. The market supply (aggregate supply) shows the total quantity of goods supplied in an economy.*

B.1. The Supply Law

$$Q_s = f(P)$$

The law of supply states that an increase in the price of a good motivates the producer to increase production and thus the quantity supplied of that good must increase. The supply curve illustrates the maximum quantity of a good sellers are willing and able to produce at each and every price, all else equal. It is a curve that slopes upward and to the right showing that as the price increases the quantity supplied increases because the good becomes more profitable and vice versa.

This positive relation can be expressed from the following linear supply equation

$$Q_s = c + dP$$

where c is the intercept of the equation and the parameter d is the slope of the supply equation as

$$Slope = \frac{\Delta\, Q_s}{\Delta\, P} = d,$$

where the positive sign confirms the supply law stated above.

B.2. The Determinants of Supply

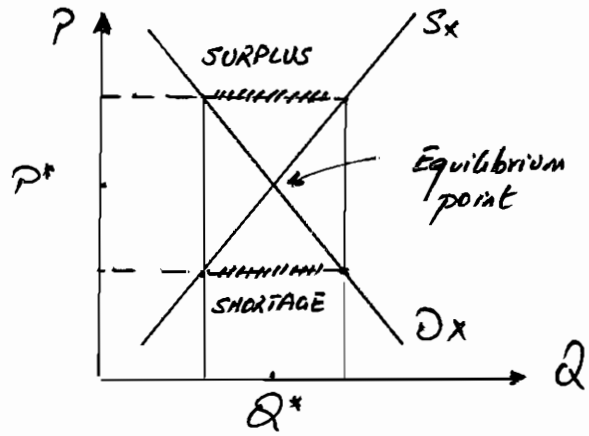
$$Q_x^s = f(\underbrace{P_x}_{+ve}, \underbrace{C, L, i, \varnothing, T}_{-ve\ -ve\ -ve\ -ve\ -ve})$$

$\underbrace{\quad}_{\text{change in } Q_s \text{ Movement}}$ $\underbrace{\quad}_{\text{Shift in the Supply curve shift the demand curve up or down}}$

where C is the cost of raw materials, L is cost of labor, i is the interest charges, $\varnothing$ denotes technology, and T is taxes. The expected signs are shown beneath the variables. The analysis of the determinants of supply is the same as the one for the determinants of demand explained above.

C. The Market Equilibrium

The intersection between demand and supply yields the equilibrium quantity and equilibrium market price. We need to distinguish between two cases and their automatic adjustments as follows:



C.1. Case of Market Surplus: $S > D$

If the actual price in the market is above the equilibrium price, then the supply exceeds the demand in the market which yields a market surplus.

The Adjustment Process (the graph is omitted) When $S > D$, the excess supply will push the price down. As price goes down, both the quantity demanded and supplied will react to this change in price such that the quantity demanded will increase (following the demand law) and the quantity supplied will decrease (following the supply law). Both changes are represented by a movement along the demand and supply curve respectively till the new equilibrium is restored.

C.2. Case of Market Shortage: $D > S$

If the actual price in the market is below the equilibrium price, then the demand exceeds the supply in the market which yields a market shortage

The Adjustment Process (the graph is omitted) When $D > S$, the excess demand will push the price up As the price goes up, both the quantity demanded and supplied will react to this change in price such that the quantity demanded will decrease (following the demand law) and the quantity supplied will increase (following the supply law) Both changes are represented by a movement along the demand and supply curve respectively till the new equilibrium is restored

Remark 4 *The government can intervene in the market and sets a maximum or a minimum price The term* ***price ceiling*** *is used when the government sets a price in the market that is below the equilibrium price Price ceilings create situations of excess demand (a shortage at the government-regulated price). On the other hand, when the government sets a price that is above the equilibrium price it is called a* ***price floor****. Price floors create excess supply (a surplus at the government-regulated price) Although price ceiling or price floor prevents the market from being in equilibrium, they are policies implemented by the government to achieve other social goals*

D. Applications: Effects of change in demand and supply on the market equilibrium

D.1. Effect of a change in demand only on equilibrium

D.2. Effect of a change in supply only on equilibrium

D.1. Effect of a change in both demand and supply on equilibrium

See separate handout

Change in D	Change in S	Effect on equilibrium P	Effect on Equilibrium Q
Increase	Increase	Indeterminate	Increase
Increase	Decrease	Increase	Indeterminate
Decrease	Decrease	Indeterminate	Decrease
Decrease	Increase	Decrease	Indeterminate

II. Elasticity

The concept of elasticity is used in many disciplines; it measures the extent to which something changes in response to something else Thus, it is after the degree of responsiveness of one variable to another The concept of elasticity can be useful for an economist who wants to measure the response of the quantity demanded or the quantity supplied for any change in the price level In what follows we shall digress on the different types of elasticities that might be useful for us

A. Price Elasticity of demand ε_d

A.1 Definition

The price elasticity of demand measures the response of the quantity demanded to any change in the price level. The more the response of the quantity demanded to the change in price, the more elastic the demand curve is and vice-versa.

$$\varepsilon_d = \frac{\% \triangle Q_d}{\% \triangle P}$$

Note that the price elasticity of demand is always negative because of the demand law, however, we express it in absolute terms as we care about its magnitude. We can have the following cases:

1. If $\% \triangle Q_d < \% \triangle P \Longrightarrow |\varepsilon_d| < 1 \Longrightarrow$ Inelastic demand
2. If $\% \triangle Q_d > \% \triangle P \Longrightarrow |\varepsilon_d| > 1 \Longrightarrow$ Elastic demand
3. If $\% \triangle Q_d = \% \triangle P \Longrightarrow \varepsilon_d = 1 \Longrightarrow$ Unit elastic demand
4. If $\% \triangle Q_d = 0 \Longrightarrow \varepsilon_d = 0 \Longrightarrow$ Perfectly Inelastic demand
5. If $\% \triangle P = 0 \Longrightarrow \varepsilon_d = \infty \Longrightarrow$ Perfectly elastic demand

(see separate handout for figures for details)

A.2. Calculations

There are two methods to calculate the elasticity; namely (1) Arc Elasticity and (2) Point Elasticity. We shall consider both methods in what follows

Arc Elasticity Arc elasticity will calculate the elasticity of demand between two points. The elasticity along the arc is calculated using the following formula

$$\varepsilon^d = \frac{\frac{(Q_2 - Q_1)}{(Q_2 + Q_1)/2}}{\frac{(P_2 - P_1)}{(P_2 + P_1)/2}}$$

Point Elasticity Point Elasticity is calculated by knowing the demand curve equation and any given point at which we can calculate the elasticity of demand. The formula used to calculate the point elasticity is

$$\varepsilon^d = \frac{\triangle Q}{\triangle P} \times \frac{P}{Q},$$

where $\frac{\triangle Q}{\triangle P}$ is slope of the demand curve

Problem 5 *Consider the following demand function*

$$Q_x^d = 60 - 2P_x$$

Calculate the elasticity of demand at a price of 10

Solution 6 *The quantity demanded at $P = 10$ is obtained by substituting P into the demand equation. This yields $Q_x^d = 40$. The price elasticity of demand is then*

$$\varepsilon^d = \frac{\triangle Q}{\triangle P} \times \frac{P}{Q} = -2 \times \frac{10}{40} = -0.5$$

or

$$|\varepsilon_d| = 0.5 < 1$$

and thus, we conclude that the demand is inelastic at $P = 10$

A.3. Elasticity and the slope of the demand curve

Although the slope of the demand curve is constant, the elasticity of demand is changing from one point to the other; moving down to the right along the demand curve, the price elasticity of demand will increase from 0 when $Q = 0$ to ∞ when $P = 0$ (see separate graph). Therefore, it is not generally true that a steep slope for a demand curve implies inelastic demand, and a flat slope for a demand curve implies elastic demand

A.4. Elasticity, Marginal Revenue, and Total Revenue

Define

$$TR = P \times Q$$

$$AR = \frac{TR}{Q}$$

$$MR = \frac{\triangle TR}{\triangle Q}$$

Note that at the mid-point of the *linear* demand curve, the price elasticity of demand equals 1. It can be shown that if

$$|\varepsilon^d| = 1 \Longrightarrow MR = 0 \Longrightarrow TR = \max$$

B. Price Elasticity of supply ε_s

B.1 Definition

The price elasticity of supply measures the response of the quantity supplied to any change in the price level. The more the response of the quantity supplied to the change in price, the more elastic the supply curve is and vice-versa

$$\varepsilon_s = \frac{\% \triangle Q_s}{\% \triangle P}$$

we can have the following cases:

1. If $\% \triangle Q_s < \% \triangle P \Longrightarrow \varepsilon_s < 1 \Longrightarrow$ Inelastic supply

2. If $\% \triangle Q_s > \% \triangle P \Longrightarrow \varepsilon_s > 1 \Longrightarrow$ Elastic supply

3. If $\% \triangle Q_s = \% \triangle P \Longrightarrow \varepsilon_s = 1 \Longrightarrow$ Unit elastic supply

4. If $\% \triangle Q_s = 0 \Longrightarrow \varepsilon_s = 0 \Longrightarrow$ Perfectly Inelastic supply

5. If $\% \triangle P = 0 \Longrightarrow \varepsilon_s = \infty \Longrightarrow$ Perfectly elastic supply

(see separate handout for figures for details)

B.2. Calculations

Same as the price elasticity of demand

C. Income Elasticity of demand I

The income elasticity of demand measures the response of the quantity demanded to any change in the income level. The more the response of the quantity demanded to the change in income, the more elastic the demand

$$I = \frac{\% \triangle Q_d}{\% \triangle Y}$$

D. Cross Elasticity of demand $\eta_{x,y}$

The cross elasticity of demand measures the percentage change of the demand for one good in response to a given percentage change in price of another good, i.e., it measures the responsiveness of the demand of our own product for a change in the price of another product (could be a complement or a substitute)

$$\eta_{x,y} = \frac{\% \triangle Q_x^d}{\% \triangle P_y}$$

$\eta_{x,y}$ is negative if x and y are perfect substitutes and positive if they are complements.

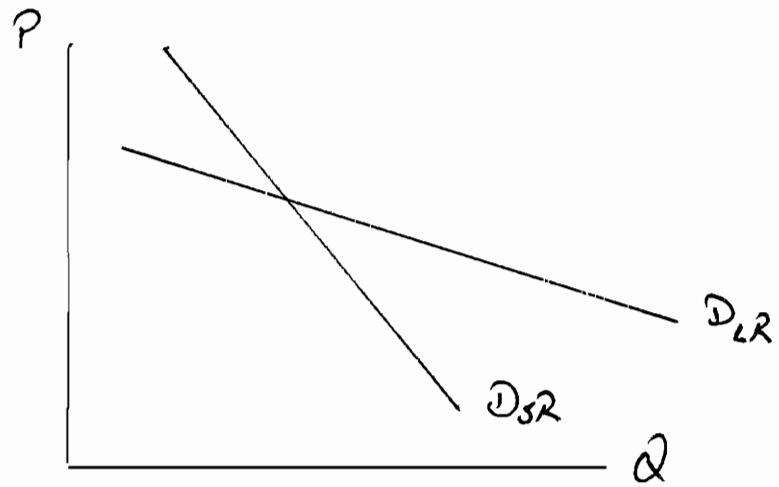
E. Short Run (SR) Versus Long Run (LR) Elasticity

We shall consider the SR and LR elasticity of demand for durable and non-durable goods as follows:

E.1. Non-durable goods (e.g., gasoline, coffee)

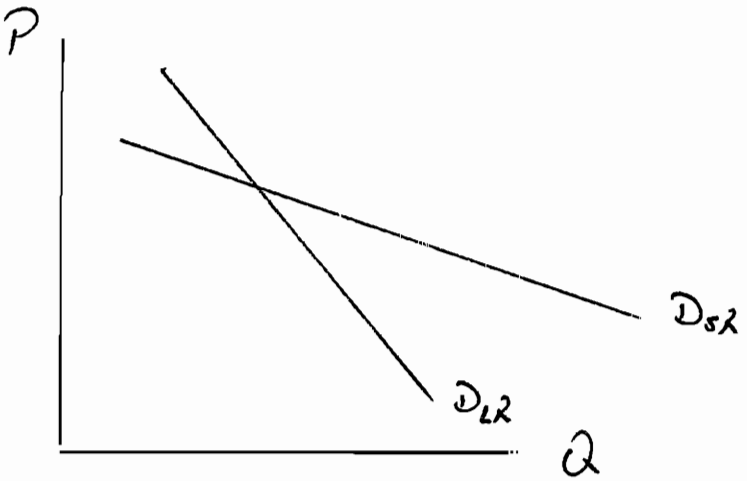
The SR demand is less elastic than the LR demand curve; that is, in the SR, the quantity demanded do not respond much to the change in prices because people can not change their consumption patterns in the short-run. In the LR, however, people can switch to other goods

and thus, the LR demand curve for these goods is more elastic in the LR (see figure below)



E.2. Durable goods (e.g., Washing machines, refrigerators, ...etc)

For this type of goods, the SR demand is more elastic than the LR demand This is because these types of goods are not purchased often and thus, in the SR, the consumer can postponed the purchase till the price decreases (see the figure below)



III. Market Failure

Market failure is a situation in which the price and the quantity that prevail in the market are not the equilibrium outcome. There are many causes of market failure as follows

1 Externalities

2. Public Goods

3. Monopoly

4. Government interventions through imposing taxes, subsidies, or quotas

The net welfare effect of market failure can be measured by the change in the consumer and producer surplus

Problem 7 *Suppose the supply of rice in the USA is*

$$Q_S = 640 + 0\,9P$$

Total demand for USA rice is comprised of domestic demand

$$Q_D(DOM) = 800 - 0\,8P$$

and export demand

$$Q_D(EXP) = 1100 - 0\,4P$$

Assume that there is no rice imported into the USA. Q is measured in million metric tonnes

(a) Calculate the equilibrium price and quantity of rice

(b) Calculate the price elasticity of export demand at equilibrium, and the price elasticity of supply at equilibrium

(c) Recently, some of the world's largest exporters of rice (Indonesia, China, India, and Vietnam) have reduced exports to secure their own domestic supplies and ensure they can feed their own citizens As a result, some of the largest importers of rice (such as the Philippines) have turned to the USA for rice, which has doubled the demand for USA rice in the export market Calculate the new equilibrium price and quantity of rice

(d) Calculate how much the export demand will change as a result

Solution:

(a) First, find the total demand as

$$Q_D(TOT) = Q_D(DOM) + Q_D(EXP) = 1900 - 1\,2P$$

At equilibrium, we have

$$Q_D(TOT) = Q_S$$

or

$$1900 - 1\,2P = 640 + 0\,9P$$

sovling for the equilibrium price P^* yields

$$P^* = \$600$$

Substitute in the demand or supply equation to obtain the equilibrium quantity

$$Q^* = 1180\ units$$

(b) The price elasticity of demand at equilibrium is obtained using the formula

$$\varepsilon^d = \frac{\triangle Q_D(EXP)}{\triangle P} \times \frac{P}{Q_D(EXP)}$$

or

$$\varepsilon^d = -0.4 \times \frac{600}{860} = -0\ 2791,$$

where at the equilibrium price

$$Q_D(EXP) = 1100 - 0\ 4(600) = 860$$

Similarly, the elasticity of supply at equilibrium is found by the following formula

$$\varepsilon^s = \frac{\triangle Q_S}{\triangle P} \times \frac{P}{Q_S} = 0\ 90 \times \frac{600}{1180} = 0.4576$$

(c)

$$\begin{aligned} Q_D(TOT)_{New} &= Q_D(DOM) + 2[Q_D(EXP)] \\ &= [800 - 0\ 9P] + 2[1100 - 0\ 4P] \\ &= 3000 - 1.6P \end{aligned}$$

The equilibrium is then

$$Q_D(TOT)_{New} = Q_S$$

or

$$3000 - 1\ 6P = 640 + 0\ 9P$$

sovling for the new equilibrium price P^*_{New} yields

$$P^*_{New} = \$944$$

Substitute in the demand or supply equation to obtain the new equilibrium quantity

$$Q^*_{New} = 1489\ 6\ units$$

(d) given the calculations above, we have

$$Q_D(EXP)_{New} = 2200 - 0\ 8(944) = 1444.8 \tag{1}$$

and

$$Q_D(EXP)_{Original} = 1100 - 0\ 4(600) = 860 \tag{2}$$

Therefore, from (1) and (2), $Q_D(EXP)$ increased by 584.80 units.

The Consumer Choice Theory

Winter, 2010 **Hany Fahmy**[1]

This lecture notes cover the basic concepts of the consumer choice theory The note is divided into two parts; part I analyze the consumer's problem by combining the consumer's ability, represented by her budget line, and the consumer's preference, represented by her indifference curve Then, the optimal choice conditions are derived and illustrated with some examples. Part II is devoted to the illustration of the consumer's demand curve, which can be derived from the consumer's optimal choice, and to the analysis of its properties in detail Multiple choice questions as well as practice problems are found in a separate handout

[1] hany fahmy@hf-consultingco com

A typical consumer combines her budget (ability) with her preference (desire) in order to choose the optimal combination of goods that maximizes her utility To study the consumer problem, we start first by analyzing the consumer's ability, represented by her budget constraint, then we discuss the consumer's preference, represented by her indifference curves.

A. The Budget Constraint

We shall consider one consumer allocating all her income m to choosing two goods, x_1 and x_2 The analysis is in the short run and thus, the prices of both goods, p_1 and p_2, are constant The budget constraint represent all possible bundles that the consumer can afford to buy The budget constraint is represented by the budget line (BL) taking the following form

$$m = p_1 x_1 + p_2 x_2$$

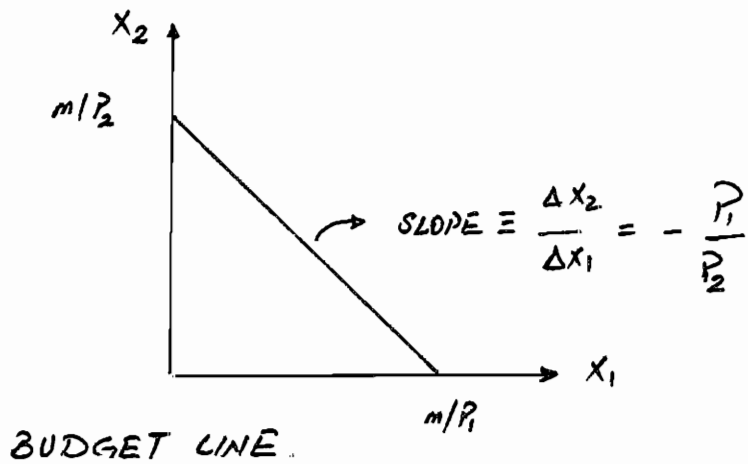
or

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1 \tag{1}$$

where

$$\frac{\triangle x_2}{\triangle x_1} = -\frac{p_1}{p_2} \equiv slope\ of\ the\ BL \tag{2}$$

Observations:



1. $\frac{m}{p_1}$ is the horizontal intercept; it represents the maximum amount of good 1 that the consumer can afford if she decides to spend all her income on good 1 only and zero on good 2

2. $\frac{m}{p_2}$ is the vertical intercept; it represents the maximum amount of good 2 that the consumer can afford if she decides to spend all her income on good 2 only and zero on good 1

3. The BL is negatively sloped, i e , the more the consumer consumes of one good, the less she has to consume from the other good, because income is limited.

4 An increase (decrease) in the price of good 1, p_1, will cause the BL to rotate inward (outward) and the slope becomes steeper (flatter)

5. An increase (decrease) in income, however, will cause a parallel shift in the BL outward (inward)

B. The Consumer's Preference

The main objective of the consumer is to maximize utility from consumption. The consumer's utility function expresses her preference over the two goods. The utility function, or the consumer's preference, is represented graphically by the indifference curve (IC) There are different types of utility functions depending on the nature of the two goods consumed, e.g , complements or substitutes. The general, well behaved[1], utility function is the Cobb-Douglas utility function and it takes the following form

$$U(x_1, x_2) = x_1^a x_2^b \tag{3}$$

where a and b are parameters. The utility function of two perfect substitutes goods takes the form

$$U(x_1, x_2) = ax_1 + bx_2 \tag{4}$$

where a and b represent the substitution proportions. Finally, The utility function of two perfect complements goods takes the form

$$U(x_1, x_2) = \min\{ax_1, bx_2\}, \tag{5}$$

where a and b are represent the complementarity proportions. The slope of the utility function is *the marginal rate of substitution* (MRS); it shows how much of good 2 should be sacrificed in order to increase the consumption of good 1 by one unit

$$MRS_{1,2} = \frac{\triangle x_2}{\triangle x_1}$$

For instance, if the $MRS = -2$, it means that in order to increase good 1 by one unit, the consumer should forgo 2 units of good 2 Note that the MRS is *negative* because the consumer has a *limited* budget. It can be easily shown (see the proof below) that the MRS can be expressed as the ratio of marginal utilities of the two goods as

$$MRS_{1\,2} = \frac{\triangle x_2}{\triangle x_1} = -\frac{MU_1}{MU_2} \tag{6}$$

where

$$MU_1 = \frac{\partial U(\cdot)}{\partial x_1}$$

and

$$MU_1 = \frac{\partial U(\,)}{\partial x_2}$$

[1]Well behaved preference menas that it satisfies all preference axioms The only utlity function that satisfies all the axioms is the Cobb-Douglas utility function; more of that alter

Proof. Since the utility level is constant along the indifference curve, $\overline{U}$, then we have

$$\overline{U} = U(x_1, x_2)$$

Total differentiate the above expression yields

$$d\overline{U} = \frac{\partial U(\ )}{\partial x_1} dx_1 + \frac{\partial U(\ )}{\partial x_2} dx_2$$

Since the derivative of a constant is zero, then we can write

$$0 = MU_1 dx_1 + MU_2 dx_2$$

or

$$0 = MU_1 \triangle x_1 + MU_2 \triangle x_2$$

Rearranging the terms yields the desired result

$$\frac{\triangle x_2}{\triangle x_1} = -\frac{MU_1}{MU_2}$$

■

TU, MU, and The Law of Diminishing Marginal Utility

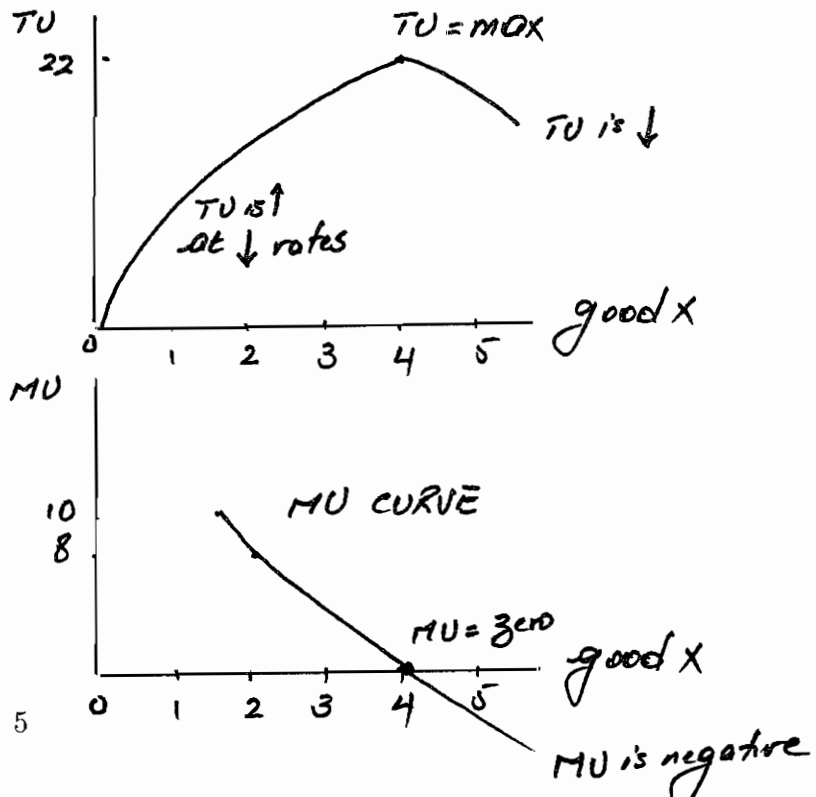
Definition 1 *The Marginal utility of a good (MU) gives the additional amount of satisfaction per additional unit consumed of that particular good*

Definition 2 *The total utility of a good (TU) gives the total satisfaction from all the unit consumed of a particular good*

The law of diminishing marginal utility is simply stating that the more the consumer increases her consumption of a good, the total utility derived from that good will be increasing at decreasing rates; that is the marginal utility of each extra unit consumed will be less than the previous one.

EXAMPLE :

GOOD X	TU_X	MU_X
1	10	10
2	18	8
3	22	4
4	22	zero
5	20	-2



As we mentioned before, any utility function is presented graphically using the IC. In what follows, we define the IC and state the basic assumptions about preferences

Definition 3 *The indifference curve shows all bundles (combinations of two goods) that give the consumer the same utility. Any point along the IC will give the consumer the same amount of satisfaction and hence, the consumer is indifferent along the IC.*

The Preference Axioms

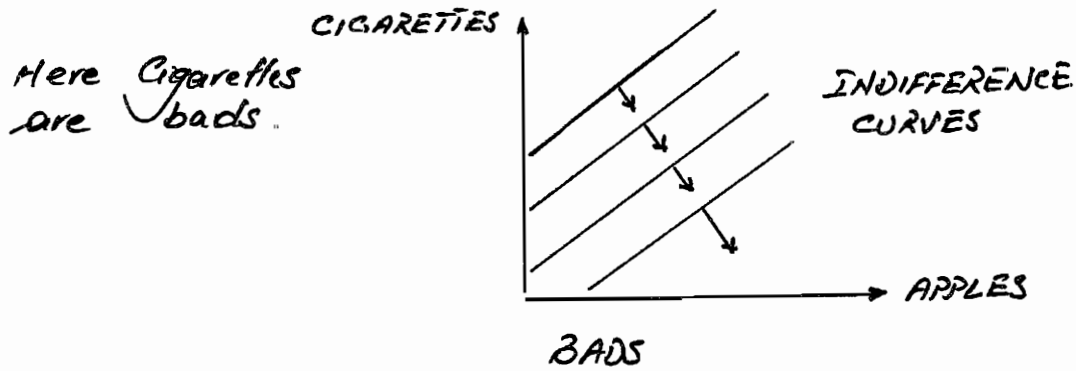
1. **Completeness.** Any preference relation should be complete in the sense that the consumer is able to compare between the bundles of goods available. Consider, for instance, the relation between the two bundles A and B. A complete relation would imply that the consumer could prefer bundle A to B, bundle B to A, or could be indifferent between both.

2. **Transitivity.** Preference relations should follow a logical order in the sense that the consumer can rank the available bundle. Consider three bundles, A, B, and C, such that the consumer prefers A to B and prefers B to C. Thus, following the transitivity assumption, A is also preferred to C.

3. **Reflexiveness.** A bundle could be at least as good as itself[2]

4. **The More is better than less.** The consumer will always be more satisfied with additional units consumed of a good (or goods) than less. In addition, consumers are never satisfied or satiated; more is always better, even just a little bit better.

   Remark 4 *Some goods may be undesirable, such as cigarettes, and consumers will always prefer less of these goods. This is illustrated in the following figure*



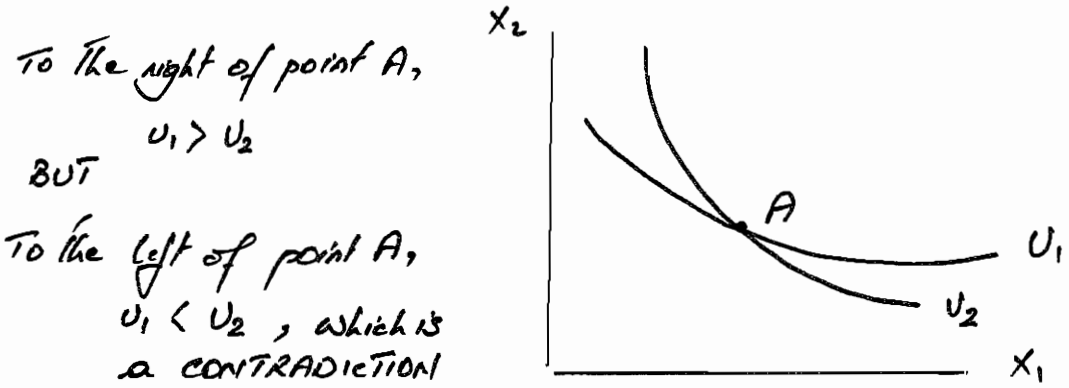
5. **Diminishing MRS.** This follows immediately from the law of diminishing marginal utility; that is the more you consume of a good the total utility will be increasing but at diminishing rate, i.e., the marginal utility is decreasing the more the consumer increases her consumption of a good.

[2]For any preference relation to exist, we require the first three axioms of preference not to be violated

Observations

1. The indifference curves will always have *negative slope*; the more the consumer increases the consumption of good 1, the less she consumes of good 2. This is because her income is limited

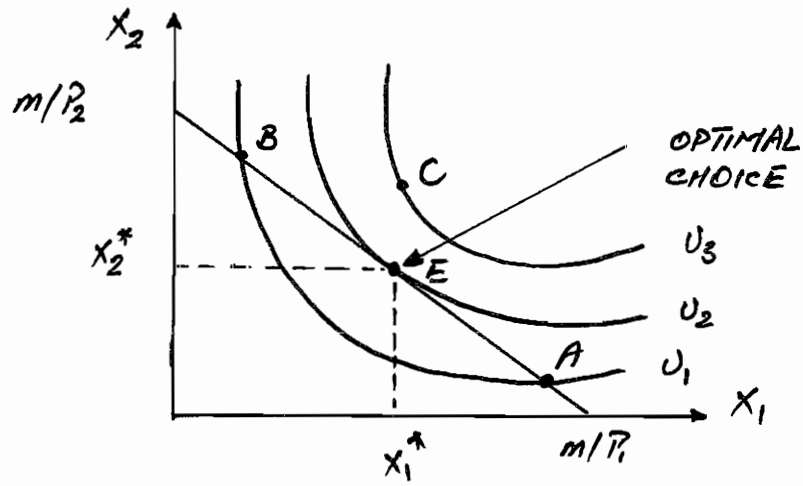
2 The indifference curves will *never intersect* Otherwise, transitivity assumption of preference is violated (see the figure below for illustration)



3 Indifference curves that satisfy all previous axioms of preferences are *convex* to the origin Actually, *diminishing marginal rate of substitution* implies convexity of the indifference curves. The more you move down and to the right along the IC, the flatter the curve will be In other words, the slope (MRS) is diminishing the more you move down and to the right towards good 1

The Optimal Choice

New let's combine the consumer's ability (the budget constraint), which is represented graphically by the budget line (equation 1), and the consumer's objective to maximize utility (preference), which is represented graphically by the IC



Observe that the optimal choice is realized at point E. Notice how E is better than A and B as it is lying on a higher indifference curve and hence the consumer enjoys more utility at E compared to A or B. Point C, however, is better than E, but it is not optimal an optimal choice because C is unattainable (un-affordable for the consumer).

The Optimality Conditions

There are two conditions that should be satisfied at optimality:

(a) The slope of the budget line is equal to the slope of the indifference curve; that is

$$\frac{MU_1}{MU_2} = \frac{p_1}{p_2}$$

and

(b) the optimal choice is on the budget line, i.e , the optimal bundle should satisfy the budget line equation

$$m = p_1 x_1 + p_2 x_2$$

Example 5 *Sasha receives utility from consuming two goods: a night out partying with her friends (F) and studying in the library for a day (S). A night out with her friends costs \$75, sasha estimates the cost of a trip to the library to study for a day to be comprised of \$10 for gas and \$10 for parking. Sasha's mother has given her an allowance of \$4000 per academic year for these two goods. Her utility function is*

$$U(F,S) = 20F^{0.2}S^{0.8}$$

Answer the following questions:

(a) Find Sasha's marginal rate of substitution of partying for studying.

(b) Calculate the quantities of nights out and days studying that Sasha should consume to maximize her utility

(c) Suppose that Sasha's mother feels she is spending too much time out with friends. Her mom limits the number of nights out to below her optimal level. Under this constraint, is her MRS of F for S greater than, less than, or equal to that at her optimal choice? Explain using a clearly labeled graph with F on the horizontal axis.

(a) Notice that Sasha's utility function is a Cobb-Douglas utility function. To find the MRS of partying for studying we first calculate the marginal utilities of both goods as

$$MU_F = \frac{\partial U(F,S)}{\partial F} = 4F^{-0.8}S^{0.8}$$

and

$$MU_S = \frac{\partial U(F,S)}{\partial S} = 16F^{0.2}S^{-0.2}$$

The MRS is therefore

$$MRS_{F,S} = \frac{MU_F}{MU_S} = \frac{4F^{-0.8}S^{0.8}}{16F^{0.2}S^{-0.2}} = \frac{S}{4F}$$

(b) To find the optimal choice, use the previously mentioned conditions as follows:

$$\frac{MU_F}{MU_S} = \frac{P_F}{P_S}$$

or

$$\frac{S}{4F} = \frac{75}{20}$$

Simple algebra yields

$$S = 15F \tag{1}$$

Next use the budget line equation

$$4000 = 75F + 20S \tag{2}$$

Now solve equation (1) and (2) simultaneously; plug (1) in (2) yields

$$4000 = 75F + 20(15F) = 3075F$$

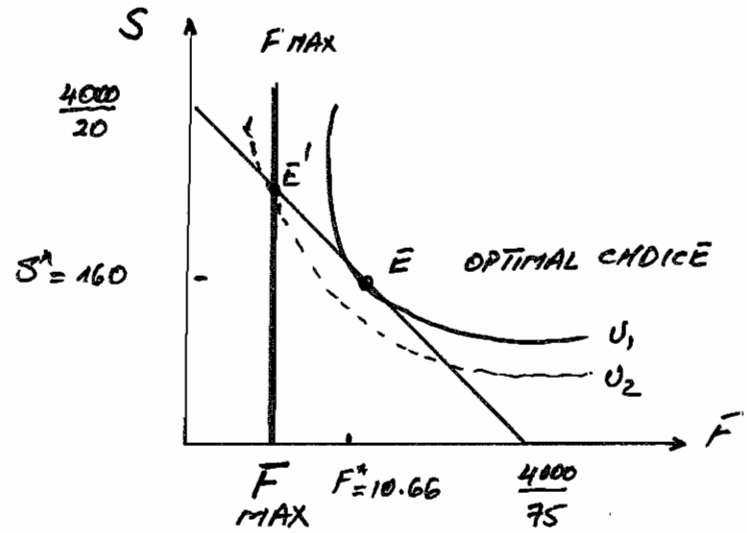
Solve for F, we have

$$F^* = 10.66 \text{ nights partying} \tag{3}$$

and plug (3) back in (1) gives

$$S^* = 15(10\ 66) = 160 \text{ days studying}$$

(c)



* THE LIMIT IS $F_{MAX} < F^*$
* NOW SASHA'S CHOICE IS AT POINT E', WHICH IS ON THE SAME BL BUT AT A LOWER IC
* WE CAN SEE THAT AT THE NEW POINT (E'), THE SLOPE OF THE IC IS STEEPER THAN AT E, i.e.,

$$MRS_{F\text{ for }S}(E') > MRS_{F\text{ for }S}(E)$$

C. Special cases of utility functions

We shall consider two special cases of utility functions In particular, we consider the case when the two goods are perfect substitutes and the case when the two goods are perfect complements

Perfect Substitutes (PS)

The utility function takes the following form

$$U(x_1, x_2) = ax_1 + bx_2$$

where a and b are the rates of substitutions. The indifference curve in such a case will be straight line and hence, the MRS will be constant.

Observations

1. The Indifference curves are straight lines
2. The MRS of good 1 for good 2 is constant
3. The consumer will choose to consume the cheaper good and therefore, the optimal choice is a boundary optimum (not interior optimum)
4. The PS utility functions violates axiom number 5 of the preference assumptions

Example 6 *Suppose that 1 cup of tea (good 1) gives John the same amount of satisfaction as 1 cup of coffee (good 2) John's utility is then*

$$U = x_1 + x_2$$

If John allocates \$50 per month on beverages and if the price of coffee is \$2 and the price of tea is \$1 5 What is John's optimal choice?

STEP 1. Find a and b as follows

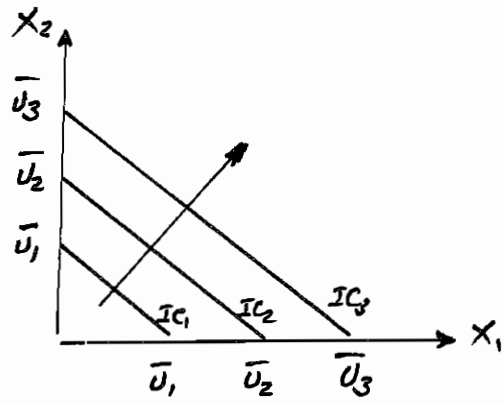
$$\frac{x_1}{x_2} = \frac{1}{1} = \frac{a}{b}$$

STEP 2. Utility function :

$$U = 1X_1 + 1X_2$$

STEP 3. Draw the IC's for any given level of utility, say $\bar{U}$, as

$$\bar{U} = X_1 + X_2 \Rightarrow X_2 = \bar{U} - X_1$$



STEP 4. Optimal choice

Since $P_1 < P_2 \Rightarrow$ John will spend all his income on good 1.

Therefore, $X_1^* = \frac{m}{P_1} = \frac{50}{1.5} = 33.3$ & $X_2^* = 0$.