

Econ 301: Assignment 1

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due: September 30, 2014, in class

Please show all the necessary calculations and explanations that lead to your answer. Provide your answers in the order the problems are given.

1. (15 points) Suppose the consumer consumes two goods: hamburgers and CDs. Her monthly income is $m = \$400$. The price of a hamburger is $p_1 = \$5$ and the price of a CD is $p_2 = \$20$.
 - a. Write down her monthly budget constraint. Illustrate graphically the budget set. Calculate the slope of the budget line.
 - b. Suppose the government levies a 25% tax on each CD the consumer purchases. What is the new monthly budget constraint? Draw the new budget line.
 - c. Now suppose that instead of the 25% tax on CDs each consumer pays a fixed sum of \$100 in taxes per month. What is the monthly budget constraint in this case? Draw the new budget line.
 - d. Now suppose that there is no tax of any kind; instead, assume that our consumer lives in a country where the government acts as a health police and rations hamburgers. In this country, each consumer is allowed to eat at most 60 hamburgers per month. What is the monthly budget constraint in this case? Draw the new budget line.
2. (10 points)
 - a. Consider a group of people and the relation "at least as old as." Is this relation transitive? Is it complete?
 - b. Consider a group of people and the relation "older than." Is this relation transitive? Is it complete?
Explain all your answers.
3. (15 points) Jon likes both mangoes and bananas. For him 2 bananas are "worth" one mango, so that he is indifferent among the bundles $(6, 0)$, $(4, 1)$, $(2, 2)$ and $(0, 3)$, for example, where each bundle is given as (x_b, x_m) , and x_b denotes the amount of bananas, while x_m denotes the amount of mangoes.
 - a. Find some bundles such that Jon is indifferent between these and the bundle $(8, 0)$.
 - b. Plot some of Jon's indifference curves in a graph.
 - c. What is the MRS for these indifference curves at different points? Do these indifference curves exhibit a diminishing marginal rate of substitution?

d. Are Jon's preferences monotonic?

4. (10 points) Determine if the two utility functions below are monotonic transformations of each other:

$$u(x_1, x_2) = 2 \ln x_1 + 3 \ln x_2$$

$$u'(x_1, x_2) = x_1^4 x_2^6$$

(Hint: Compute the marginal rate of substitution for both and see if they are the same. Think about why this method works when we want to recognize preferences that are represented by different utility functions.)

5. (25 points) Art is a history major. He has to take two courses, one in philosophy and the other in mathematics. He is relatively good at philosophy, but not so much at math. He needs to spend 1 hour studying for philosophy in order to improve his grade by one point, while he needs to spend 2 hours studying for math in order to do the same. Art has a total of 60 hours to study for the two subjects. Suppose that he would get 50 points in each course if he didn't study at all.

a. We want to analyze Art's problem with consumption theory. What are the goods, prices, and income? Write down the budget constraint. Illustrate it with a graph. (Hint: consider the *improvements* on Art's grades. That is, the goods are improvements in philosophy and math, and the prices are the cost of improvement measured in hours.)

b. Art's satisfaction from his academic record is solely determined by the lower of his grades in philosophy and mathematics. For example, he is indifferent between 60 in philosophy and 70 in math, and 60 in philosophy and 60 in math. What kind of preferences does Art have over improvements in his philosophy and mathematics grades? Find a utility representation for these preferences. Derive his optimal choice of grades. Illustrate it in a graph.

c. Now suppose that Art has to pass both subjects by obtaining at least 60 points in both courses. Assuming that this is a "budget restriction," illustrate Art's new budget set in a graph. What is Art's optimal choice in this case?

6. (25 points) Rosalie spends all her income on (pairs of) shoes (good 1) and books (good 2). Her utility function can be represented by $u(x_1, x_2) = x_1 x_2$. Her income is $m = \$200$. The prices are $p_1 = \$20$ and $p_2 = \$10$.

a. Derive Rosalie's optimal consumption bundle. Show the complete calculation, and illustrate your answer graphically (draw the indifference curves and the budget constraint).

b. How would your answer change to part a. if Rosalie's utility function were given by $u(x_1, x_2) = x_1^2 x_2^2$?

- c. What is Rosalie's marginal utility of shoes when she buys 2 pairs of shoes and 5 books? Compute this marginal utility for both utility functions given above.
- d. Now suppose the government levies a 25% tax on each pair of shoes purchased. Derive Rosalie's consumption bundle and illustrate it in the same graph that you drew in part a.
- e. Would Rosalie prefer a \$20 income tax (i.e., lump-sum tax) to the 25% value tax on shoes? Which scenario would the government prefer? Explain your answers.