1 $m = 400$ $P_1 = 5$ $P_2 = 20$

$P_1X_1 + P_2X_2 = m$

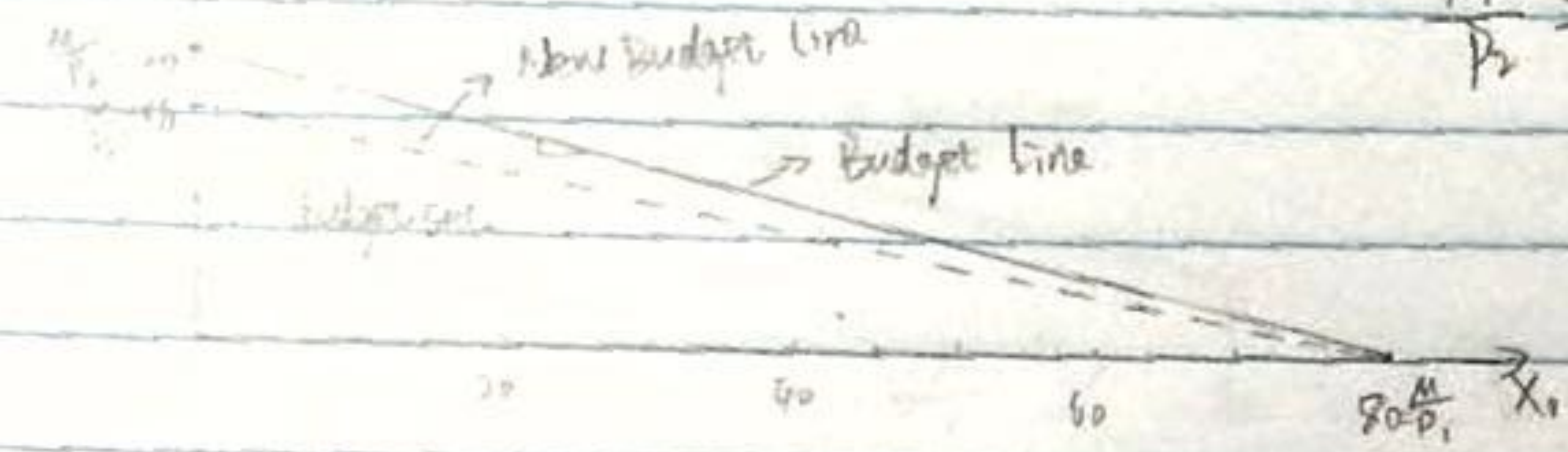
$5X_1 + 20X_2 = 400$ (Budget constraint)

$$\frac{M}{P_1} = \frac{400}{5} = 80$$

$$\frac{M}{P_2} = \frac{400}{20} = 20$$

$$\text{slope: } \frac{-\frac{M}{P_2}}{\frac{M}{P_1}} = \frac{-P_1}{P_2} = -\frac{5}{20}$$

$$\text{slope} = -\frac{1}{4}$$



b. $t = 25\% = 0.25$ on each CD.

so $P_2' = (1+0.25)P_2 = 1.25P_2 = 1.25 \cdot 20 = 25$

$\Rightarrow$ New Budget Constraint $5X_1 + 25X_2 = 400$

$\frac{M}{P_1} = \frac{400}{5} = 80$ $\frac{M}{P_2'} = \frac{400}{25} = 16$

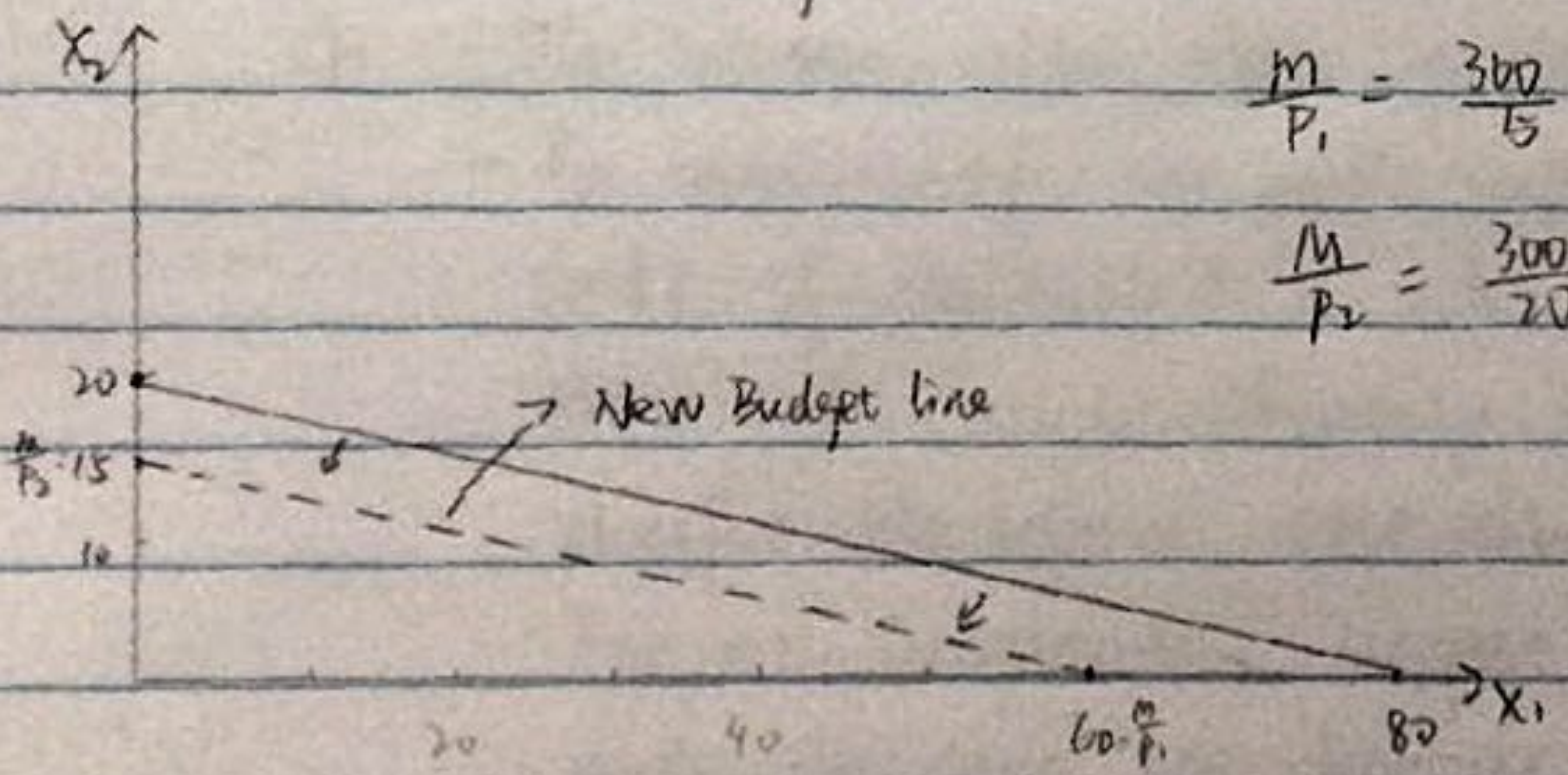
c. a fixed sum in tax $= 100$.

so $m' = 400 - 100 = 300$

$\Rightarrow$ New Budget Constraint $5X_1 + 20X_2 = 300$

$$\frac{m}{P_1} = \frac{300}{5} = 60$$

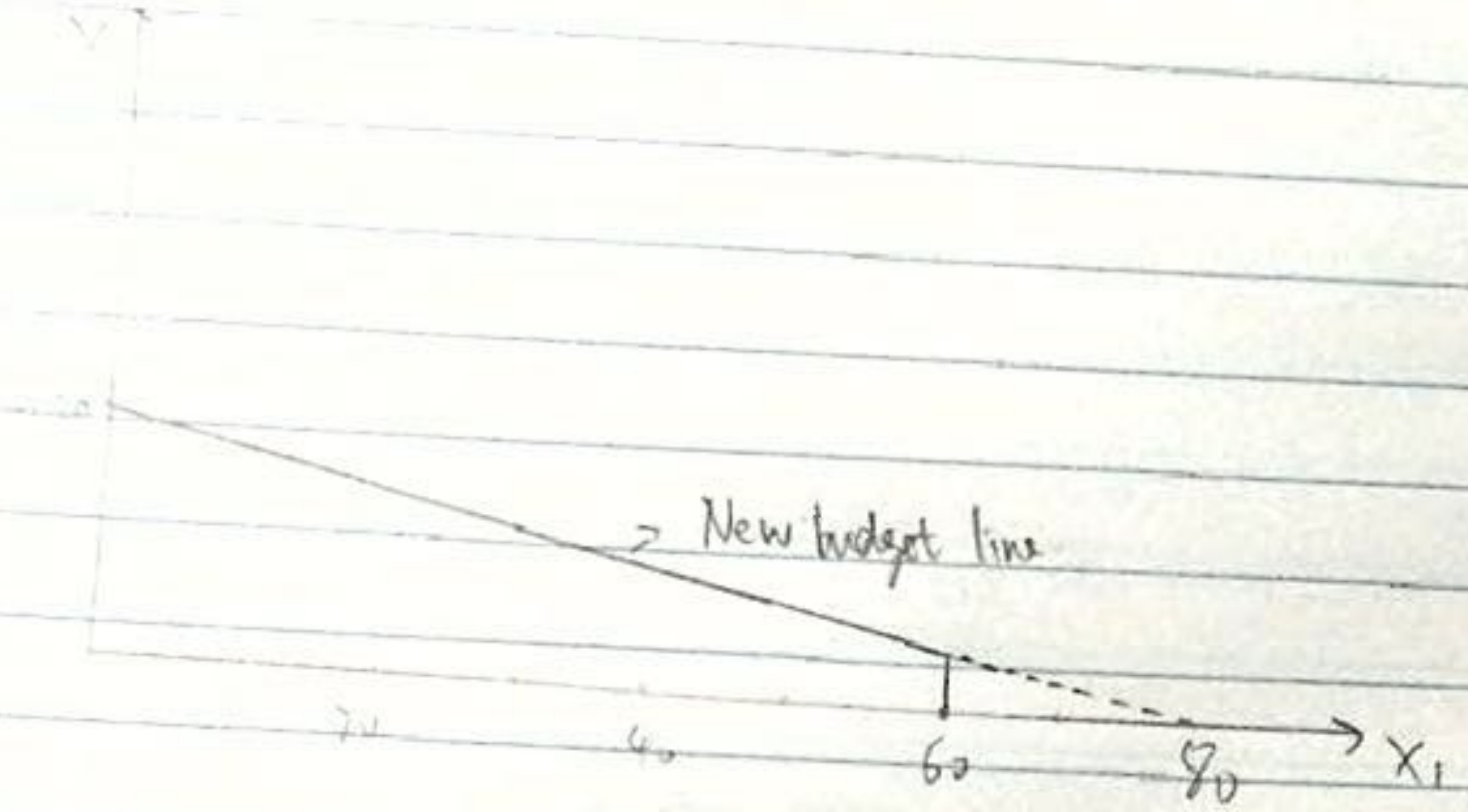
$$\frac{M}{P_2} = \frac{300}{20} = 15$$



d. $m = 400$ $P_1 = 5$ $P_2 = 20$

because no more than 60

New budget line is $5x_1 + 20x_2 = 400$ $(x_1 \le 60)$



2. a The relation is transitive and complete

— For transitivity, if $A > B$. $B > C$. so $A > C$, therefore, it is transitive

For complete, no matter what age we choose, we can find at least as old as itself.

b. the relation is transitive and complete

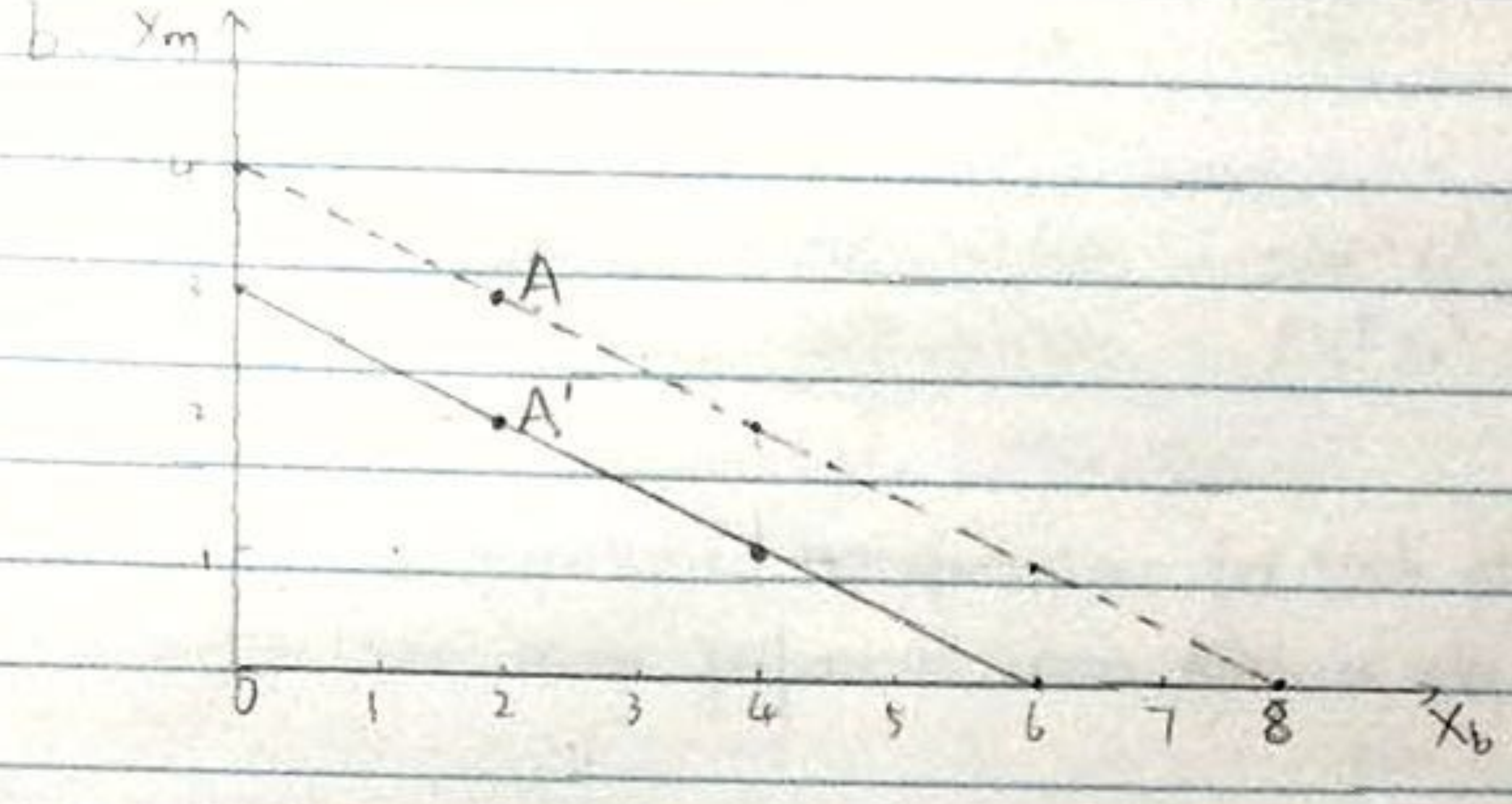
— For transitivity. if A is old than B. B is older than C, so A is old than C.

— For complete. we can chose a smaller age than the original age

For example. pick $34 < 35$ so it is complete

3. a. given 2 bananas = 1 mango and the bundle (8,0).
so we can find some bundles which are (6,1), (4,2), (2,3), and (0,4).

b.



c. $MRS_{bm} = \frac{\Delta X_m}{\Delta X_b} = \frac{4-3}{8-6} = \frac{1}{2}$

As the graph above shows, these indifference curves doesn't exhibit a diminishing marginal rate of substitution because MRS at any point on the indiffarent curve are identical.

d. Yes. Jon's preferences are monotonic.
Monotonicity implies "more is better".
If $X_1 \geq Y_1$, and $X_2 \geq Y_2$ with at least one strict inequality,
then $X(X_1, X_2) > Y(Y_1, Y_2)$.
For example, see the points in the graph above.
$A(2,3)$, $A'(2,2)$
$2=2$, and $3>2$, then $A(2,3) > A'(2,2)$
therefore, Jon's preferences are monotonic.

4. $u(x_1, x_2) = 2\ln x_1 + 3\ln x_2$

$u'(x_1, x_2) = x_1^4 x_2^6$

$$MRS = \frac{-MU_1}{MU_2} = -\frac{\partial u(x_1,x_2)/\partial x_1}{\partial u(x_1,x_2)/\partial x_2} = \frac{-2/x_1}{3/x_2} = -\frac{2}{3}\frac{x_2}{x_1}$$

$$MRS = \frac{-MU_1}{MU_2} = -\frac{\partial u(x_1,x_2)/\partial x_1}{\partial u(x_1,x_2)/\partial x_2} = \frac{-4x_1^3 x_2^6}{6x_1^4 x_2^5} = -\frac{2}{3}\frac{x_2}{x_1}$$

u and u' are monotonic transformations for each other because u and u' represent the same preferences when MRS is the same for u and u'.

5. a. Income = 60 hours

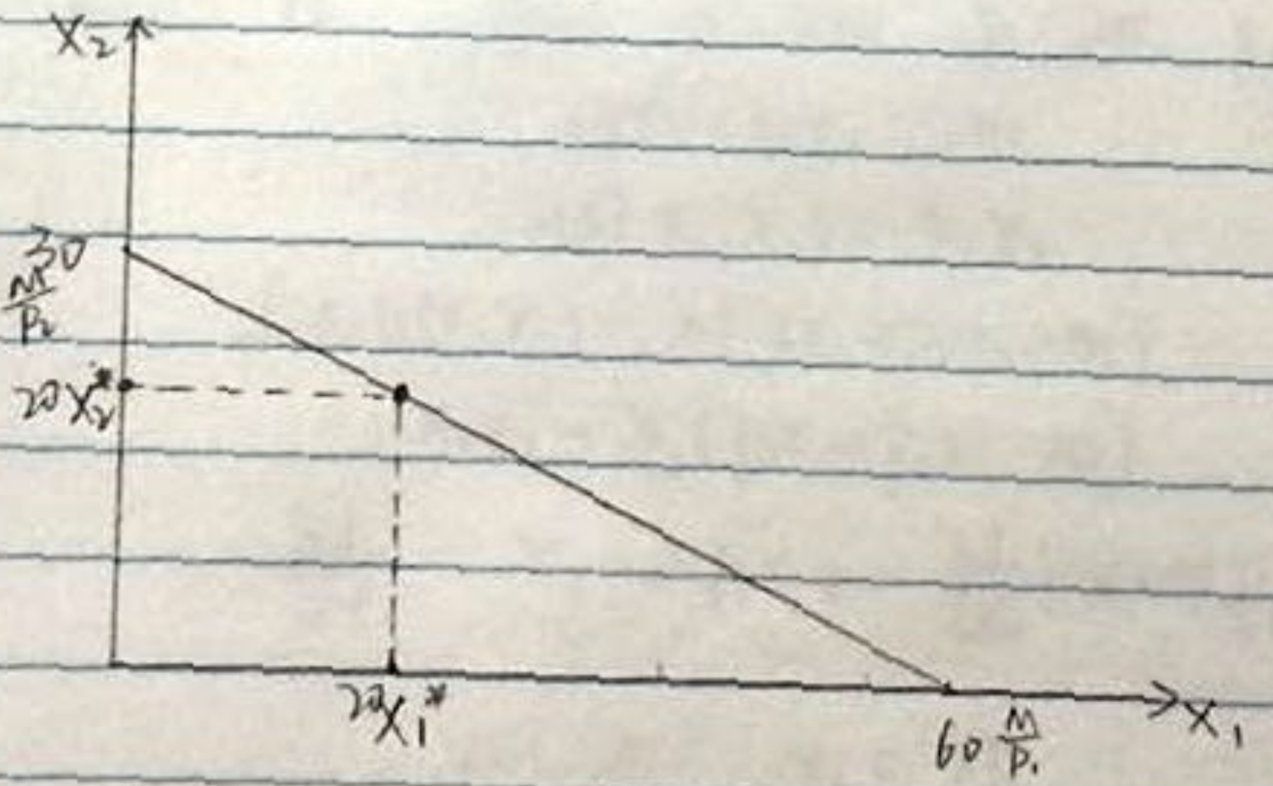
good 1 = 1 point improvement in philosophy

good 2 = 1 point improvement in math

P_1 = 1 hour

P_2 = 2 hours

Budget Constraint $x_1 + 2x_2 = 60$



b. The preferences for Art is perfect complement.

The utility function is $u(x_1, x_2) = \min\{x_1, x_2\}$

The optimal bundle is $x_1^* = x_2^*$

so. $x_1 + 2x_2 = 60$

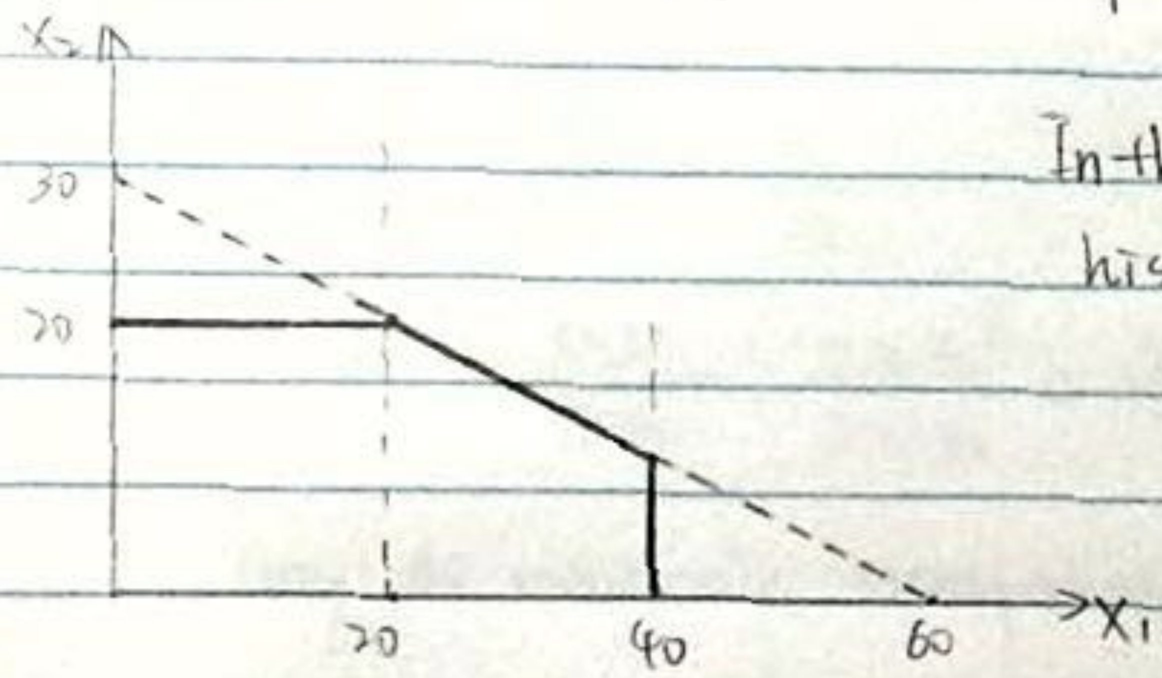
$x_1^* + 2x_2^* = 60$

$3x_1^* = 60$

$x_1^* = 20 \quad x_2^* = 20$

The grades for Art in philosophy is $50 + x_1^* = 70$ and in math is $50 + x_2^* = 70$ because studying one hour on philosophy and two hours on math Art can get 1 point on philosophy and math, so Art will spend 20 hours on studying philosophy and 40 hours on studying math if he wants to get 70 on both courses.

c. given at least 60 points to pass both courses.
that means he need to improve both courses by 10 points.
so he needs to spend at least 10 hours on philosophy to get the 10 points and spend at least 20 hours on math to get the 10 points.



In this case, Art's optimal choice does not change. his optimal bundle still is (20, 20)

6. a. given $m=200$, $p_1=20$, $p_2=10$

$$20x_1+10x_2=200$$

$$u(x_1,x_2)=x_1x_2$$

$$\mathcal{L}=u(x_1,x_2)-\lambda[p_1x_1+p_2x_2-m]$$

$$=x_1x_2-\lambda[p_1x_1+p_2x_2-m]$$

$$\frac{\partial\mathcal{L}}{\partial x_1}=x_2-\lambda p_1=0 \quad \frac{\partial\mathcal{L}}{\partial x_2}=x_1-\lambda p_2=0$$

$$x_2=\lambda p_1 \quad x_1=\lambda p_2$$

$$\frac{p_1}{p_2}=\frac{x_2}{x_1}$$

$$x_2=\frac{p_1}{p_2}x_1 \Rightarrow$$

$$p_1x_1+p_2x_2=m$$

$$p_1x_1+p_2\cdot\frac{p_1}{p_2}x_1=m$$

$$p_1x_1+p_1x_1=m$$

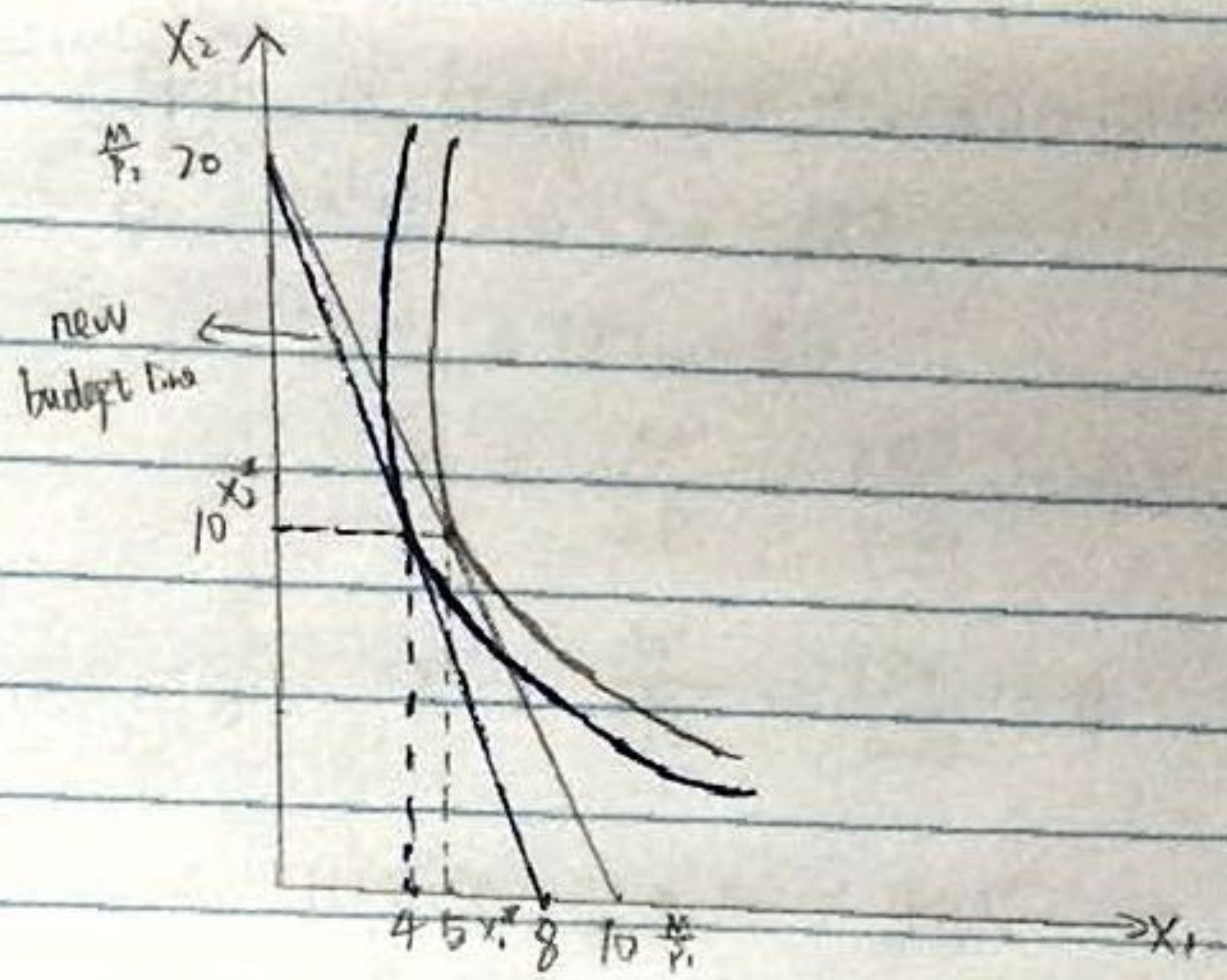
$$2p_1x_1=m$$

$$x_1=\frac{m}{2p_1}$$

$$x_2=\frac{m}{2p_2}$$

$$x_1=\frac{200}{2\cdot 20}=5$$

$$x_2=\frac{200}{2\cdot 10}=10$$



b $\quad u(X_1, X_2) = X_1^2 X_2^2$

$$MRS = \frac{MU_1}{MU_2} = \frac{X_2}{X_1}$$

$$MRS' = \frac{MU_1}{MU_2} = \frac{2X_1X_2^2}{2X_1^2X_2} = \frac{X_2}{X_1}$$

They are monotonic transformations for each other

c. $\quad$ given $X_1 = 2$, $X_2 = 5$

$$MU_1 = \frac{du}{dX_1} = X_2 \qquad MU_1' = \frac{du}{dX_1} = 2X_1X_2^2$$

$$MU_1 = 5 \qquad MU_1' = 2 \cdot 2 \cdot 5^2 = 100$$

d $\quad$ given a 25% tax on shoes.

$P_1' = (1+25\%)P_1 = 1.25 \cdot 20 = 25$

from a we know $\quad X_1 = \frac{M}{2P_1}$

$X_2 = \frac{M}{2P_2}$

so. $\quad X_1' = \frac{M}{2P_1'} = \frac{200}{2 \cdot 25} = 4$

X_2 doesn't change $X_2 = 10$.

e. given a lump-sum tax \$20

so. $M' = 200 - 20 = 180$.

$20X_1' + 10X_2' = 80$

$$X_1' = \frac{M'}{2P_1} = \frac{180}{2 \cdot 20} = 4.5$$

$$X_2' = \frac{M'}{2P_2} = \frac{180}{2 \cdot 10} = 9$$

$$u'(X_1', X_2') = X_1'X_2' = 40.5$$

know from d. $u(X_1, X_2) = X_1X_2 = 4 \cdot 10 = 40$

Therefore, Rosalie would prefer to lump-sum tax.

For government, lump-sum $= 20$

value tax $= 25\% \cdot 20 \cdot 4 = 20$

so. government is indifferent with two different tax.