

MAT1374 Probability and Games of Chance: Poker101 Midterm Solutions

1. Which of the following best describes the difference between sequential games and simultaneous games?
 - A. In sequential games players have many strategies, while in simultaneous games there are only two strategies available.
 - B. In sequential games players take turns making moves, while in simultaneous games players make their move without knowing what the others will do.
 - C. Sequential games are more complicated to analyse.
 - D. Sequential games involve two players, while simultaneous games involve many players.

Answer: B. Both sequential and simultaneous games can involve any number of players and strategies, and can range from trivial to highly complex.

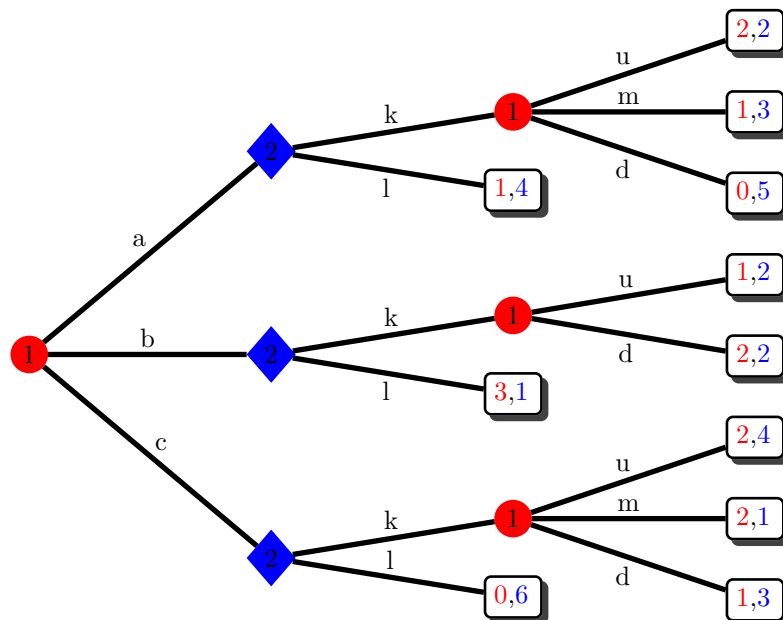
2. Which of the following is NOT a good example of a sequential game?
 - A. Chess
 - B. Rock-Paper-Scissors
 - C. Tic-Tac-Toe

Answer: B. In RPS both players decide at the same moment, while in the other games you wait for the opponent's move.

3. Which of the following statements about rollback equilibria in sequential games is correct?
 - A. In a rollback equilibrium all players receive the highest possible payoff.
 - B. In a rollback equilibrium at least one of the players receives the highest possible payoff.
 - C. In a rollback equilibrium the sum of the payoffs of all the players is maximized.
 - D. None of the above.

Answer: D. A rollback equilibrium may give a low payoff to one or even all players.

4. Consider the following game tree:



How many rollback equilibria are there in this game?

- A. None.
- B. One, namely (a,k,u).
- C. One, namely (b,k,d).
- D. One, namely (c,k,u).
- E. Two, namely (a,k,u) and (b,k,d).

Answer: C: Backward induction reveals that (b,k,d) is the only equilibrium.

5. What game is this?

		Player 2	
		a	b
Player 1	a	12 12	20 0
	b	0 20	3 3

- A. Chicken
- B. Battle of the Sexes
- C. Prisoner's Dilemma

Answer: C. You can tell from the fact that both players have a dominant strategy (and from the fact that the NE isn't Pareto optimal).

6. Consider the following 2x2 game matrix:

		Player 2	
		a	b
Player 1	a	1 0	3 2
	b	0 2	2 1

Which of the following is true?

- A. Neither player has a dominant strategy.
- B. Player 1 has a dominant strategy, player 2 doesn't.
- C. Player 2 has a dominant strategy, player 1 doesn't.
- D. Both players have a dominant strategy.

Answer: C. Strategy b is dominant for player 2, but player 1 doesn't have a dominant strategy.

7. Consider again the 2x2 game matrix from the previous question. Which of the following is true?

- (A) There are no Nash Equilibria in pure strategies.
- (B) There is one Nash Equilibrium, namely (a,b).
- (C) There is one Nash Equilibrium, namely (b,b).

(D) There are two Nash Equilibria, namely (a,b) and (b,b).

Answer: B. Player 2 plays the dominant strategy b, and Player 1 will play the best response to that, namely a.

8. Which of the following statements about Nash Equilibria in a 2-player game is NOT true?

- A. In a Nash Equilibrium, both players have correct beliefs about what the other will play.
- B. In a Nash Equilibrium, no player can improve his/her payoff by unilaterally changing strategies.
- C. In a Nash Equilibrium, both players receive the best possible payoff.
- D. In a Nash Equilibrium, each player acts in his/her own best interest.

Answer: C. In for example the prisoner's dilemma, the NE gives the second worst payoff to both players.

9. We typically measure the outcomes of a game or a bet in terms of utility. Which of the following is NOT a good argument for the claim that it is not accurate to measure the outcomes only in terms of money won or lost?

- A. Some people make irrational decisions when they are blinded by monetary gain or loss.
- B. For some people the prestige of winning is more important than the financial gain or loss.
- C. Different people attach different values to a certain financial gain or loss.
- D. Social and cultural values and standards may influence the way people play in certain games.

Answer: A. All the other answers are good reasons why looking at money alone is too narrow to determine what rational behavior is.

10. What is a *Tragedy of the Commons*?

- A. A situation where a large group of people act irrationally, with a negative outcome for society as a result. Example: the riots in Vancouver after the hockey games.
- B. A situation where a small group of people make a rational move which has negative consequences for a large part of the population. Example: subprime mortgage crisis.
- C. A situation where everyone acts in their own best interest, but with a negative outcome for society as a result. Example: traffic congestion.
- D. A situation where everyone thinks they are acting in the best interest of society, but where this backfires. Example: giving too much power to an evil leader.

Answer: C. (See notes.)

11. Which of the following best describes what Cardano's Law is about?

- A. It tells how to determine the likelihood that the outcome of a random process such as throwing a die is favorable.
- B. It indicates how to predict future behavior of a random process from past observations.
- C. It explains why certain processes such as throwing a die are inherently unpredictable.
- D. It is a tool for determining whether a process such as flipping a coin is biased or genuinely random.

Answer: A. (See notes.)

12. Suppose we throw two fair dice. What is the probability that the sum of the dice is 5?

- A. 1/36
- B. 2/36
- C. 4/36
- D. 5/36

Answer: C. Out of 36 possible outcomes, 4 are favorable, namely (1,4),(2,3),(3,2),(4,1).

13. What is the expected value of the following bet? We flip a fair coin twice. If it comes heads at least one time, I pay you \$4. Otherwise you pay me \$8.

- A. -\$4
- B. \$1
- C. \$2
- D. \$4

Answer: B.

$$EV = \frac{3}{4} \cdot \$4 + \frac{1}{4} \cdot (-\$8) = \$1.$$

14. What do the so-called *paradoxes in rationality* demonstrate?

- A. That the axioms of utility theory are often violated by subjects.
- B. That utility is not a useful concept for understanding human decision making.
- C. That there is no hope of giving a quantitative theory of rational decision making.
- D. That humans are often incapable of correctly applying utility theory to everyday decision making.

Answer: A. (See lecture 6.)

15. What is the difference between a type 1 and a type 2 Gambler's Fallacy?

- A. The type 1 fallacy denies that you can predict future outcomes based on past patterns, while the type 2 fallacy believes you can use such patterns to predict the future.
- B. The type 2 fallacy centers around the idea that streaks of particular outcomes are likely to continue in the future, while according to the type 1 fallacy such streaks are likely to be offset with opposite outcomes.
- C. The type 1 fallacy asserts that certain random experiments are biased, while the type 2 fallacy asserts that even though these experiments are not biased, we can still predict future outcomes based on patterns.
- D. The type 1 fallacy denies the validity of the law of large numbers, while the type 2 fallacy asserts that the law is correct, but that it doesn't apply to many random experiments such as flipping coins or throwing dice.

Answer: B. (See notes.)

Part 2: Long Answer Questions

Question 1. Consider the following game:

		Player 2		
		p	q	r
Player 1	a	$\begin{matrix} \text{blue } 2 \\ \text{red } 2 \end{matrix}$	$\begin{matrix} \text{blue } 1 \\ \text{red } 3 \end{matrix}$	$\begin{matrix} \text{blue } -1 \\ \text{red } 1 \end{matrix}$
	b	$\begin{matrix} \text{blue } 2 \\ \text{red } 0 \end{matrix}$	$\begin{matrix} \text{blue } 2 \\ \text{red } 0 \end{matrix}$	$\begin{matrix} \text{blue } -1 \\ \text{red } -1 \end{matrix}$
	c	$\begin{matrix} \text{blue } 1 \\ \text{red } 3 \end{matrix}$	$\begin{matrix} \text{blue } -1 \\ \text{red } 1 \end{matrix}$	$\begin{matrix} \text{blue } 0 \\ \text{red } 2 \end{matrix}$

Find all Nash Equilibria in this game. Explain clearly what method you use and how you apply this.

Answer: You can use dominant/dominated strategies, or best responses. If you use IEDS, first remove the dominated strategy b. (This is the only possibility.) Then observe that p is dominant. So remove q,r. Finally, c is dominant, so remove a. That leaves us with (c,p) as NE.

Note: at the beginning p is weakly dominant, but that doesn't mean you can remove r and s; you can only do that after removing b.

If you use best responses, you find that the best response to a is p, to b is p or q, to c is p, and that the best response to p is c, to q is a and to r is c. Thus p and c are mutual best responses, hence a NE.

To get full marks, you must (a) choose one of these methods, (b) correctly apply the method, (c) explain how you apply the method and (d) draw the correct conclusion. Only saying: "I used IEDS, removed the dominated strategies and found (c,p)" is not sufficient.

Note: the singular is "equilibrium", the plural is "equilibria".

Question 2. Explain, using the concept of Expected Value, why it is inaccurate to judge whether a bet is good or not by just looking at the probability that you win the bet. Your answer should include an illustrative example. Use 200 words maximum.

Answer: The key point is that just looking at probabilities ignores the crucial aspect of payoffs; this is exactly what EV takes into account, as by definition EV is the weighted average of the payoffs, weighted by their probabilities. As such, it expresses the average payoff associated to a bet: if this quantity is positive, then the bet is good; otherwise it is bad.

As an example, you should give a bet which has a high ($> 50\%$) probability of being won, but a negative EV. For example, in roulette you can bet on red and black at the same time, giving you a $36/37$ chance of winning. Even though this chance is very high, you still have a negative EV (as calculated in class and in the notes), so this is still not a good bet.

Alternatively, you can give an example of a bet which has a very small chance of being won but which has positive EV.

To get full marks you need to explain clearly that EV takes the crucial information about payoffs into account, and why that makes EV into a more useful criterion than mere probability. Your example should be one which demonstrates clearly that a judgment based purely on probability can be incorrect, while one based on EV different and correct.

Note: the question is NOT about the difference between monetary payoffs and utility. Also, considerations about playing a single round versus playing multiple rounds of the same game are not relevant. Even if you play just one round the concept of EV is the correct one to use.

Part 3: Multiple Choice Questions

1. What is an astralagus?
 - A. An early version of the die, made out of the heelbone of animals.
 - B. An early version of the playing card, made out of parchment.
 - C. A device for predicting the movement of celestial bodies.
 - D. A calculating device used for determining probabilities.

Answer: A. (See notes.)

2. What was the main reason for the decline of the popularity of game theory after 1960?
 - A. Game theory had failed to give useful strategic advice concerning the nuclear arms race.
 - B. The main military and strategic problems of the time had been solved, and most game theorists started working on more theoretical matters.
 - C. The general perception was that game theory portrayed a cynical, calculating image of human beings and did nothing positive to address the pressing problems of that time.
 - D. There were insufficient employment opportunities for game theorists at that time, so most of them gave up on the subject.

Answer: C. (See notes.)

3. Which of the following is a key difference between a standard martingale strategy and a reverse martingale strategy?
 - A. The standard martingale strategy aims at maximizing winnings, while the reverse martingale aims at minimizing losses.
 - B. The standard martingale strategy requires a much smaller working capital because of the smaller chance of going broke.
 - C. The standard martingale strategy recommends betting on the same outcome after a loss, while the reverse martingale strategy recommends betting on the opposite outcome after a loss.
 - D. The standard martingale strategy presents a small chance of incurring a large loss, while the reverse martingale strategy presents a large chance of incurring a small loss.

Answer: D. With a standard martingale you win small amounts most of the time, while occasionally you incur a big loss when you lose many spins in a row. The reverse is just the opposite: you lose a small amount most of the time, but you win big once in a blue moon.

4. Roulette (European). Suppose I bet \$10 on black and \$10 on 0. What is the expectation of this bet?
 - A. $-\$10$
 - B. $-\$ \frac{20}{37}$
 - C. $-\$ \frac{10}{37}$
 - D. $-\$ \frac{2}{37}$
 - E. $-\$ \frac{1}{37}$.

Answer: B. The expectation of a bet in roulette is always $-1/37$ times your bet amount.

5. Texas Hold'em. You have a drawing hand, and you have a 40% chance to make the winning hand on the river. The pot is \$35. Your opponent (who currently has the best hand) now bets his last \$10 into the \$35 pot. Which of the following is the best reasoning? (Assume this is the last action in the hand, and no betting will take place on the river.)

- A. Call, because the size of the pot is larger than the size of the bet.
- B. Call, because the expectation of the call is higher than the expectation of folding.
- C. Fold, because the chance of winning the hand is too small.
- D. Fold, because the expectation of the call is negative.

Answer: B.

$$EV = 0.4 \cdot \$45 + 0.6 \cdot (-\$10) > 0.$$

Since the EV is positive (i.e. higher than 0, the EV of folding) you should call.

6. What is the St. Petersburg Paradox meant to show?
- A. That games with infinite payoffs are practically impossible.
 - B. That expected utility cannot be equated with expected monetary payoff.
 - C. That certain bets can be deceptive and counterintuitive.
 - D. That bets involving high payoffs and small chances of winning are usually considered unattractive by subjects.

Answer: B. The expected monetary payoff is infinite, but the bet doesn't have infinite expected utility.

7. What is the defining characteristic of a sucker bet?
- A. It offers less than even money.
 - B. You have less than 50% chance of winning it.
 - C. It will only be accepted when you're sure to lose.
 - D. It has negative expectation.

Answer: C. (See notes.)

8. Which of the following statements about the battle of the sexes game is correct?
- A. Because neither player has a dominant strategy there are no Nash Equilibria.
 - B. No player has a dominant strategy, but there is exactly one Nash Equilibrium.
 - C. Both players have a dominant strategy, and there is exactly one Nash Equilibrium.
 - D. No player has a dominant strategy, and there are two Nash Equilibria.

Answer: D. (See notes.)

9. Consider the following variation of the game "21 flags" as shown in class: there are 25 flags, and each turn a player may remove 1,2,3 or 4 flags. The player who takes the last flag loses. What is the winning first move?
- A. Take 1 flag.
 - B. Take 2 flags.
 - C. Take 3 flags.
 - D. Take 4 flags.

Answer: D. If you leave your opponent with 1 flag you win. This you can do when you are left with 2,3,4 or 5 flags. Thus if you leave your opponent with 6 flags you win. Generally, you must leave your opponent with $5k + 1$ flags to win.

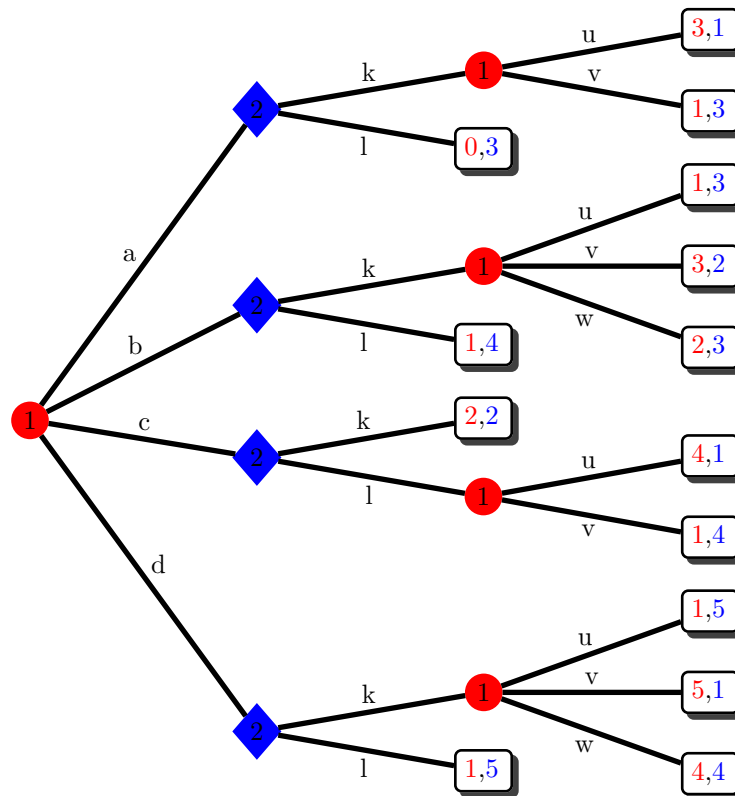
10. In class, we played the “Public Investment Game”, where you had to decide how much to invest (0-10 chips) and where the payouts depended only on the overall investment. Which of the following best describes the nature of that game?
- A. This is a multi-person prisoner’s dilemma
 - B. This is a multi-person coordination game
 - C. This is a multi-person battle of the sexes game
 - D. This is a multi-person chicken game

Answer: A. Everyone has a dominant strategy, namely investing 0.

11. What do the game-theoretic experiments involving the Ultimatum Game show?
- A. That most subjects are incapable of finding the optimal strategy in this game.
 - B. That subjects often diverge from the optimal strategy out of considerations of fairness or other social norms.
 - C. That most subjects are driven more by desire for financial gain than by a desire to find the correct strategy.
 - D. That subjects often diverge from rational play because they realize that they are part of an experiment.

Answer: B. (See notes.)

12. Consider the following game tree.



Which of the following is correct?

- A. There are no rollback equilibria.

- B. There is one rollback equilibrium, namely (b,k,v)
- C. There is one rollback equilibrium, namely (b,k,w)
- D. There is one rollback equilibrium, namely (d,k,w)
- E. There is one rollback equilibrium, namely (c,k)
- F. There are two rollback equilibria, namely (b,k,v) and (d,k,w).

Answer: E. Using backward induction we see that (c,k) is the only equilibrium.

13. Gerolamo Cardano is known for his contributions to algebra and probability theory. What was his profession?

- A. Professor of Law.
- B. Physician.
- C. Astronomer.
- D. Politician.

Answer: B. (See notes.)

14. Texas Hold'em. You are holding $A\clubsuit K\clubsuit$, and your opponent has $T\diamondsuit T\heartsuit$. The board on the turn is $2\clubsuit 2\diamondsuit 6\clubsuit 7\heartsuit$. What is the probability that the river card gives you the winning hand?

- A. 8/44
- B. 9/44
- C. 14/44
- D. 15/44

Answer: C. You have the following outs:

$$3\clubsuit, 4\clubsuit, 5\clubsuit, 7\clubsuit, 8\clubsuit, 9\clubsuit, J\clubsuit, Q\clubsuit, K\clubsuit, K\diamondsuit, K\heartsuit, A\spadesuit, A\diamondsuit, A\heartsuit.$$

Thus you win in 14/44 cases.

15. Consider the following game matrix:

		Player 2		
		p	q	r
Player 1	a	1 0	0 2	2 1
	b	3 2	4 1	0 1
	c	3 1	0 0	2 2

Which of the following is true?

- A. There are no Nash Equilibria in pure strategies.
- B. There is one Nash Equilibrium, namely (b,p).

- C. There is one Nash Equilibrium, namely (c,r).
- D. There are two Nash Equilibria, namely (b,p) and (c,r).
- E. None of the above.

Answer: A. (Use best responses.)

16. What is prospect theory?

- A. A version of game theory which takes possible future actions by players into account.
- B. A psychological theory about future actions of individuals based on their past behavior.
- C. A theory of preferences and decision making which focuses on the fact that humans are sensitive to perceived gains and losses in wealth.
- D. A theory of rationality based on the concept that humans anticipate potential regret when making decisions.

Answer: C. (See lecture 6.)

17. What was John von Neumann's role in the development of game theory?

- A. His 1928 article on the theory parlor games laid out the mathematical foundations of game theory.
- B. He proved that all games have Nash Equilibria.
- C. His 1944 book with Oskar Morgenstern *Theory of Games and Economic Behavior* was the first text to consider the competitive nature of the free market.
- D. He used game theory to prove that the US needed to expand its nuclear arsenal in order to get through the Cold War unscathed.

Answer: A. (See notes.)

18. In a scene of the movie "A Beautiful Mind", John Nash and his friends are in a bar and are eyeing a group of girls. The blonde is the most desirable, but Nash reasons that if they all try to woo her, none of them will succeed; moreover, they will then also fail with her friends, because none of them likes to be second choice. Therefore, he concludes, the best solution is when all of them ignore the blonde and each approaches one of her friends. Is this strategy a Nash Equilibrium?

- A. Yes, because all players (guys) end up with a satisfactory outcome.
- B. Yes, because the alternatives don't give a better outcome.
- C. No, because nobody achieves the best possible outcome (being with the blonde)
- D. No, because each guy would have an incentive to cheat and go for the blonde while the others go for her friends.

Answer: D. If one of the players can unilaterally improve his payoffs, then it's not a NE.

19. Thomas Schelling performed an experiment in which he had pairs of mutual strangers try to meet each other somewhere in New York City. What was the purpose of this experiment and what can be concluded from the results?

- A. The purpose was to find out how adept people are at navigating maps under conditions of time pressure and uncertainty. The conclusion was that most people are good at it.
- B. The purpose was to find out whether people with intimate knowledge of the layout of a city act differently in this task than people who had never visited the city before. The conclusion was that there is no difference.

- C. The purpose was to find out whether people can do better than guessing randomly by using focal points. The conclusion was that people rely heavily on such focal points.
- D. The purpose was to find out whether people use game-theoretically optimal strategies in complex search tasks. The conclusion was that most people fall short of complete rationality.

Answer: C. (See notes.)

20. What is the difference between normative and descriptive game theory?
- A. Normative game theory investigates how games should be studied, while descriptive game theory investigates how science actually approaches the study of games.
 - B. Normative game theory solves games using mathematics, while descriptive game theory solves games using non-mathematical techniques.
 - C. Normative game theory is only concerned with 2-player games, while descriptive game theory studies games between large groups of people.
 - D. Normative game theory looks for optimal strategies, while descriptive game theory tries to understand why people play the way they do in practice.

Answer: D. (See notes.)

BONUS QUESTION [1 point] Fries are a very popular snack in Holland. What is the most popular topping for fries?

- A. Salt and vinegar
- B. Mayonnaise
- C. Ketchup
- D. Hot sauce

Answer: B. Salt and vinegar is unheard of in Holland. The Dutch do use hot sauce occasionally, but certainly not on fries. Both ketchup and mayonnaise are used, but mayonnaise is by far the most common topping.