

Assignment 10: Thermodynamics I: Ideal Gas Law, Thermal Expansion, Calorimetry

Released: Friday Nov 23

Due Friday Nov 30, 2PM

- 1 An automobile tire is inflated with air originally at 10.0°C and normal atmospheric pressure. During the process, the air is compressed to 28.0% of its original volume and the temperature is increased to 40.0°C. (a) What is the tire pressure? (b) After the car is driven at high speed, the tire air temperature rises to 85.0°C and the interior volume of the tire increases by 2.00%. What is the new tire pressure (absolute) in pascals?

(a) Initially, $P_i V_i = n_i R T_i$ $(1.00 \text{ atm}) V_i = n_i R (10.0 + 273.15) \text{ K}$
 Finally, $P_f V_f = n_f R T_f$ $P_f (0.280 V_i) = n_i R (40.0 + 273.15) \text{ K}$

Dividing these equations,

$$\frac{0.280 P_f}{1.00 \text{ atm}} = \frac{313.15 \text{ K}}{283.15 \text{ K}}$$
 giving $P_f = 3.95 \text{ atm}$
 or $P_f = \boxed{4.00 \times 10^5 \text{ Pa (abs.)}}$.

(b) After being driven $P_d (1.02)(0.280 V_i) = n_i R (85.0 + 273.15) \text{ K}$
 $P_d = 1.121 P_f = \boxed{4.49 \times 10^5 \text{ Pa}}$

2. a) Just 9.00 g of water is placed in a 2.00-L pressure cooker and heated to 500°C. What is the pressure inside the container?
 b) 1 mole of ideal gas has total internal energy of 200J. What is the average kinetic energy (in eV) of one molecule of this gas?

a)
$$P = \frac{nRT}{V} = \left(\frac{9.00 \text{ g}}{18.0 \text{ g/mol}} \right) \left(\frac{8.314 \text{ J}}{\text{mol K}} \right) \left(\frac{773 \text{ K}}{2.00 \times 10^{-3} \text{ m}^3} \right) = \boxed{1.61 \text{ MPa}} = 15.9 \text{ atm}$$

b)
$$E = \frac{E_{\text{int}}}{N_A} = \frac{200 \text{ J}}{6.02 \times 10^{23}} = 33.22 \times 10^{-23} \text{ J} = 20.76 \times 10^{-4} \text{ eV}$$

- 3 A certain telescope forms an image of part of a cluster of stars on a square silicon charge-coupled detector (CCD) chip 2.00 cm on each side. A star field is focused on the CCD when it is first turned on and its temperature is 20.0°C. The star field contains 5 342 stars scattered uniformly. To make the detector more sensitive, it is cooled to -100°C. How many star images then fit onto the chip? The average coefficient of linear expansion of silicon is $4.68 \times 10^{-6} (\text{°C})^{-1}$.

The area of the chip decreases according to $\Delta A = \gamma A_i \Delta T = A_f - A_i$ $A_f = A_i (1 + \gamma \Delta T) = A_i (1 + 2\alpha \Delta T)$

The star images are scattered uniformly, so the number N of stars that fit is proportional to the area.

Then $N_f = N_i (1 + 2\alpha \Delta T) = 5\,342 \left[1 + 2(4.68 \times 10^{-6} \text{ °C}^{-1})(-100\text{°C} - 20\text{°C}) \right] = \boxed{5\,336 \text{ star images}}$.

- 7 A 5g lead bullet travelling with $v=1200\text{m/s}$ embeds in the 1kg block of ice at (-1°C). Knowing that practically all of the kinetic energy of the bullet is converted to heat find the final temperature of the bullet + ice system. How much water was produced during the melting?

$$K = \frac{1}{2} m v^2 = 3600 \text{ J} \quad \text{but} \quad Q(T=1 \rightarrow T=0) = m c_{\text{ice}} \Delta T = 2108 \text{ J}$$

$$K - Q = L \Delta m \Rightarrow 1492 \text{ J} = 334000 \text{ J} \Delta m \Rightarrow \Delta m = 4.5 \text{ g}$$

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Assignment 10: Cont.

- 4 100 grams of oxygen and 100 grams of hydrogen gas occupy separate, equal sections of 200 liter tank. The divide is removed and the gases are allowed to mix and react with each other. The temperature is kept constant at 110 °C, throughout the process
- find the pressure of each gas in the separate containers
 - find the pressure after the reaction ends.

$$pV = nRT \Rightarrow p = \frac{nRT}{V} \quad n(H_2) = 50 \text{ moles} \quad n(O_2) = 3.125 \text{ moles}$$

$$p(O_2) = \frac{3.125(8.31)(383)}{0.1} = 99460 \text{ Pa} \quad p(H_2) = \frac{50(8.31)(383)}{0.1} = 1591365 \text{ Pa}$$

Since the basic reaction is given by : $2H_2 + O_2 \rightarrow 2H_2O$ there will be 6.25 moles of steam and 43.75 moles of hydrogen left

$$pV = nRT \Rightarrow p = \frac{nRT}{V} \quad n(H_2) = 50 \text{ moles} \quad n(O_2) = 3.125 \text{ moles}$$

$$p(H_2O) = \frac{6.25(8.31)(383)}{0.2} = 99460 \text{ Pa} \quad p(H_2) = \frac{43.75(8.31)(383)}{0.2} = 696222 \text{ Pa}$$

$$p_{fin} = p(H_2O) + p(H_2) = 795682 \text{ Pa}$$

- 5 The mass of a hot-air balloon and its cargo (not including the air inside) is 200 kg. The air outside is at 10.0°C and 101 kPa. The volume of the balloon is 400 m³. To what temperature must the air in the balloon be heated before the balloon will lift off? (Air density at 10.0°C is 1.25 kg/m³.)

$$\sum F_y = 0: \quad \rho_{out}gV - \rho_{in}gV - (200 \text{ kg})g = 0$$

$$(\rho_{out} - \rho_{in})(400 \text{ m}^3) = 200 \text{ kg}$$

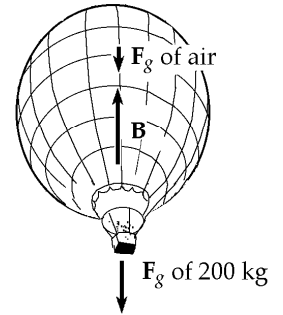
The density of the air outside is 1.25 kg/m³.

$$\text{From } PV = nRT, \quad \frac{n}{V} = \frac{P}{RT}$$

The density is inversely proportional to the temperature, and the density of the hot air is

$$\rho_{in} = \left(1.25 \text{ kg/m}^3\right) \left(\frac{283 \text{ K}}{T_{in}}\right) \quad \text{Then } \left(1.25 \text{ kg/m}^3\right) \left(1 - \frac{283 \text{ K}}{T_{in}}\right) (400 \text{ m}^3) = 200 \text{ kg}$$

$$1 - \frac{283 \text{ K}}{T_{in}} = 0.400 \quad 0.600 = \frac{283 \text{ K}}{T_{in}} \quad T_{in} = \boxed{472 \text{ K}}$$



- 6 A 1kg of ice at -30° C is added to 10 kg of steam at 500°C. answer the following questions:
- What is the final phase of the system of ice + steam if no heat escaped from it.
 - What is the final temperature when the equilibrium is established

ANS: a) final phase is steam

b) 10 kg of steam will cool down to temperature T' while the 1kg of ice is melted to water heated to 100C and converted to steam at 100C. We will find this T' from the following equations:

$$Q_1 = (10 \text{ kg})c_{steam}(T' - 500)$$

$$Q_2 = (1 \text{ kg})c_{ice}(0 - (-30)) + (1 \text{ kg})L_{melt} + (1 \text{ kg})c_{water}(100 - 0) + (1 \text{ kg})L_{evap}$$

$$Q_1 + Q_2 = 0$$

once this process is finished we have "two different steams" mixing : 1kg of steam at 100C and 10 kg at T'.

it just simple mixing: $(10 \text{ kg})c_{steam}(T_f - T') + (1 \text{ kg})c_{steam}(T_f - 100) = 0$