SOLUTIONS

1) Ralph Edmund loves steak and potatoes. Therefore, he has decided to go on a steady diet of only these two foods for all his meals. Ralph realizes that this is not the healthiest diet, so he wants to make sure that he eats the right quantities of the two foods to satisfy some key nutritional requirements. He has obtained the following nutritional and costs data:

Grams of Ingredient Per Serving.

Ingredient	Steak	Potato	Daily Requirement (g)
Carbs	5	15	>=50
Protein	20	5	>=40
Fat	15	2	>=60
Cost/Serving	$4	$2	

Ralph wishes to determine the number of daily servings of steak and potatoes that will meet these requirements at a minimum cost. Formulate the Linear programming model for this problem.

Let S **= servings of steak in diet**
P **= servings of potatoes in the diet**

Minimize $C = \$4S + \$2P$,

subject to $5S + 15P \geq 50$
$20S + 5P \geq 40$
$15S + 2P >= 60$

and $S \geq 0, P \geq 0$.

2) The Oak Works is a family owned business that makes hand crafted dining room tables and chairs. They obtain the oak from a local tree farm, which ships them 2500 pounds of oak each month. Each table uses 50 pounds of oak while each chair uses 25 pounds of oak. The family builds all the furniture itself and has 480 hours of labour available each month. Each table or chair requires 6 hours of labour. Each table nets Oak Works \$400 in profit, while each chair nets them \$100 in profit. Since chairs are often sold with tables they want to produce at least twice as many chairs as tables. Formula a linear program to maximize profit.

Let T **= # of tables to produce**

C **= # of chairs to produce**

Maximize $P = \$400T + \$100C$

subject to $50T + 25C \leq 2{,}500$ **-> Raw Material Constraint**

$6T + 6C \leq 480$ **-> Labour Constraint**

$C \geq 2T$ **->** ***Management Constraint***

and $T \geq 0$, $C \geq 0$ **-> Non Negativity Constraint**

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes—large, medium, and small—that yield a net unit profit of \$420, \$360, and \$300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

3) Let x_{ij} = number of units produced at plant i of product j (i = 1, 2, 3; j = L, M, S).

Maximize Profit = \$420($x_{1L} + x_{2L} + x_{3L}$) + \$360($x_{1M} + x_{2M} + x_{3M}$) + \$300($x_{1S} + x_{2S} + x_{3S}$)

subject to

$x_{1L} + x_{1M} + x_{1S} \le 750$ -> Capacity P1
$x_{2L} + x_{2M} + x_{2S} \le 900$ -> Capacity P2
$x_{3L} + x_{3M} + x_{3S} \le 450$ -> Capacity P3

$20x_{1L} + 15x_{1M} + 12x_{1S} \le 13{,}000$ square feet
$20x_{2L} + 15x_{2M} + 12x_{2S} \le 12{,}000$ square feet
$20x_{3L} + 15x_{3M} + 12x_{3S} \le 5{,}000$ square feet

$x_{1L} + x_{2L} + x_{3L} \le 900$ –> Demand for L
$x_{1M} + x_{2M} + x_{3M} \le 1{,}200$ –> Demand for M
$x_{1S} + x_{2S} + x_{3S} \le 750$ -> Demand for S

$(x_{1L} + x_{1M} + x_{1S}) / 750 = (x_{2L} + x_{2M} + x_{2S}) / 900$ –> Same Proportion of Capacity Constraints
$(x_{1L} + x_{1M} + x_{1S}) / 750 = (x_{3L} + x_{3M} + x_{3S}) / 450$ –> Same Proportion of Capacity Constraints

and

$x_{1L} \ge 0, x_{1M} \ge 0, x_{1S} \ge 0, x_{2L} \ge 0, x_{2M} \ge 0, x_{2S} \ge 0, x_{3L} \ge 0, x_{3M} \ge 0, x_{3S} \ge 0.$

4)

Slim-Down Manufacturing makes a line of nutritionally complete, weight-reduction beverages. One of their products is a strawberry shake that is designed to be a complete meal. The strawberry shake consists of several ingredients. Some information about each of these ingredients is given below.

Ingredient	*Calories from Fat (per tbsp.)*	*Total Calories (per tbsp.)*	*Vitamin Content (mg/tbsp.)*	*Thickeners (mg/tbsp.)*	*Cost (¢/tbsp.)*
Strawberry flavoring	1	50	20	3	10
Cream	75	100	0	8	8
Vitamin supplement	0	0	50	1	25
Artificial sweetener	0	120	0	2	15
Thickening agent	30	80	2	25	6

The nutritional requirements are as follows. The beverage must total between 380 and 420 calories (inclusive). No more than 20 percent of the total calories should come from fat. There must be at least 50 milligrams (mg) of vitamin content. For taste reasons, there must be at least two tablespoons (tbsp.) of strawberry flavoring for each tbsp. of artificial sweetener. Finally, to maintain proper thickness, there must be exactly 15 mg of thickeners in the beverage.

Management would like to select the quantity of each ingredient for the beverage that would minimize cost while meeting the above requirements.

Let

S **= Tablespoons of strawberry flavoring,**
CR **= Tablespoons of cream,**
V **= Tablespoons of vitamin supplement,**
A **= Tablespoons of artificial sweetener,**
T **= Tablespoons of thickening agent,**

Minimize $C = \$0.10S + \$0.08CR + \$0.25V + \$0.15A + \$0.06T$

subject to:

$50S + 100CR + 120A + 80T \geq 380$ **calories -> Minimum total Calories**

$50S + 100CR + 120A + 80T \leq 420$ **calories –> Maximum Total Calories**

$(S + 75CR + 30T)/(50S + 100C + 120A + 80T) \leq 0.2$

$20S + 50V + 2T \geq 50$ **mg Vitamins**

$S \geq 2A$

$3S + 8CR + V + 2A + 25T = 15$ **mg Thickeners**

and

$S \geq 0, CR \geq 0, V \geq 0, A \geq 0, T \geq 0.$

5)

	B	C	D	E	F	G	H
3		TV Spots	Magazine Ads	SS Ads			
4	Exposures per Ad	1,300	600	500			
5	(thousands)						
6					Budget		Budget
7			Cost per Ad ($thousands)		Spent		Available
8	Ad Budget	300	150	100	4,000	2	4,000
9	Planning Budget	90	30	40	1,000	2	1,000
10							
11							Total Exposures
12		TV Spots	Magazine Ads	SS Ads			(thousands)
13	Number of Ads	0	20	10			17,000
14		2					
15	Max TV Spots	5					

Let x1=#of TV Spots
Let x2=#of Magazine Ads
Let x3=#of SS Ads

MAX Exposures = 1300*x1 + 600*x2 + 500*x3

Subject to: 300*x1+150*x2+100*x3<=4000

90*x1+30*x2+40*x3<=1000

x1 >=0, x1 >=0, x1 >=0

x1<=5

Consider the Super Grain Corp. case study as presented in Section 3.1, including the spreadsheet in Figure 3.1 showing its formulation and optimal solution. Use the Excel Solver to generate the sensitivity report. Then use this report to independently address each of the following questions.

a. How much could the total number of expected exposure units be increased for each additional $1,000 added to the advertising budget?

b. Your answer in part *a* would remain valid for how large of an increase in the advertising budget?

c. How much could the total number of expected exposure units be increased for each additional $1,000 added to the planning budget?

d. Your answer in part *c* would remain valid for how large of an increase in the planning budget?

e. Would your answers in parts *a* and *c* definitely remain valid if *both* the advertising budget and planning budget were increased by $100,000 each?

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C13	TVSpots	0	-50	1300	50	1E+30
D13	Number of Ads Magazine Ads	20	0	600	150	50
E13	Number of Ads SS Ads	10	0	500	300	33.333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F8	Ad Budget Spent	4,000	3	4000	1000	1500
F9	Planning Budget Spent	1,000	5	1000	600	200

a) **Look at the Ad Budget Spent Row. The total number of expected exposures could be increased by 3,000 (shadow price) for each additional $1,000 added to the advertising budget. Notice that the units are in thousands.**

b) **Look at the Ad Budget Spent Row. This shadow price remains valid for increases of up to $1,000,000. (Again in thousands of dollars)**

c) **Look at the Planning Budget Row. The total number of expected exposure units could be increased by 5,000 (shadow price) for each additional $1,000 added to the planning budget.**

d) **This remains valid for increases of up to $600,000.**

e) **Percentage of allowable increase for ad budget = (100) / 1,000 = 10%**
Percentage of allowable increase for planning budget = (100) / 600 = 16.7%
The sum is 26.7% ≤ 100%, so the shadow prices are still valid.

****iF WE WERE ASKED TO DECREASE BOTH CONSTRAINTS SIMULTANEOUSLY BY 200, YOU WOULD DO THE FOLLOWING.**

AD Budget – Deacrease by 200,000. 200/1500 = 0.1333

Planning Budget – Deacrease by 200,000 . 200/200=1

Total Change = 1+0.1333 = 1.1333 or 113% >100%, shadow prices do not remain valid. Optimal Solution Changes.

6)

Four cargo ships will be used for shipping goods from one port to four other ports (labeled 1, 2, 3, 4). Any ship can be used for making any one of these four trips. However, because of differences in the ships and cargoes, the total cost of loading, transporting, and unloading the goods for the different ship–port combinations varies considerably, as shown in the following table:

	Port			
	1	*2*	*3*	*4*
Ship				
1	$500	$400	$600	$700
2	600	600	700	500
3	700	500	700	600
4	500	400	600	600

The objective is to assign the four ships to four different ports in such a way as to minimize the total cost for all four shipments.

Let x_{ij} = number of ships i travelling to port j (i = 1, 2, 3, 4; j = 1, 2, 3, 4).

Minimize Cost = $\$500\, x_{11} + \$400\, x_{12} + \$600\, x_{13} + \$700\, x_{14} + \$600\, x_{21} + \$600\, x_{22} + \$700\, x_{23} + \$500\, x_{24} + \$700\, x_{31} + \$500\, x_{32} + \$700\, x_{33} + \$600\, x_{34} + \$500\, x_{41} + \$400\, x_{42} + \$600\, x_{43} + \$600\, x_{44}$

subject to

$x_{11} + x_{12} + x_{13} + x_{14} = 1$

$x_{21} + x_{22} + x_{23} + x_{24} = 1$

$x_{31} + x_{32} + x_{33} + x_{34} = 1$

$x_{41} + x_{42} + x_{43} + x_{44} = 1$

$x_{11} + x_{21} + x_{31} + x_{41} = 1$

$x_{12} + x_{22} + x_{32} + x_{42} = 1$

$x_{13} + x_{23} + x_{33} + x_{43} = 1$

$x_{14} + x_{24} + x_{34} + x_{44} = 1$

and $x_{11} \geq 0, x_{12} \geq 0, x_{13} \geq 0, x_{14} \geq 0, x_{21} \geq 0, x_{22} \geq 0, x_{23} \geq 0, x_{24} \geq 0, x_{31} \geq 0, x_{32} \geq 0, x_{33} \geq 0, x_{34} \geq 0, x_{41} \geq 0, x_{42} \geq 0, x_{43} \geq 0, x_{44} \geq 0$.

	A	B	C	D	E	F	G	H	I
1	**Unit Cost**			Port					
2			1	2	3	4			
3		1	$500	$400	$600	$700			
4	Ship	2	$600	$600	$700	$500			
5		3	$700	$500	$700	$600			
6		4	$500	$400	$600	$600			
7									
8									
9	**Assignments**			Port			Total		
10			1	2	3	4	Assignments		Supply
11		1	0	1	0	0	1	=	1
12	Ship	2	0	0	0	1	1	=	1
13		3	0	0	1	0	1	=	1
14		4	1	0	0	0	1	=	1
15	Total Assigned		1	1	1	1			
16			=	=	=	=			Total Cost
17	Demand		1	1	1	1			$2,100

Sample Transportation Problem Set up:

	1	2	3	4	5				Supply
1						0	=		15
2						0	=		20
3						0	=		15
	0	0	0	0	0				
=	=	=	=	=					
Demand	11	12	9	10	8			Min C	0

	1	2	3	4	5
1	61	72	45	55	66
2	69	78	60	49	56
3	59	66	63	61	47