University of Ottawa
PHY1321/1331
Principles of Physics
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CLASS: PHY_____

STUDENT #: ____________________

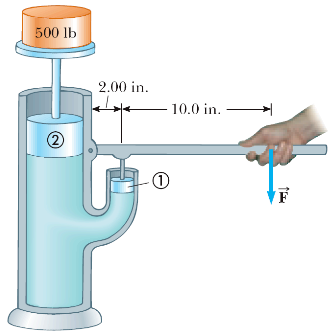
NAME: ______________________

Assignment 9: FLUIDS

Released: Saturday Nov 18 11:30AM Due: **Friday Nov 23 2 PM**

1 Review problem. Piston (1) in Figure P15.11 has a diameter of 0.250 in. Piston (2) has a diameter of 1.50 in. Determine the magnitude *F* of the force necessary to support the 500 lb load in the absence of friction.

CHECK PG 4



2 A spherical aluminum ball of mass 1.26kg contains empty spherical cavity that is concentric with the ball. The ball just barely floats in water. Calculate (a) the outer radius of the ball and (b) the radius of the cavity

SOLUTION:

(a) The weight of the ball must be equal to the buoyant force of the water:

$$1.26\ \mathrm{kg}\,g = \rho_{\mathrm{water}}\frac{4}{3}\pi r_{\mathrm{outer}}^3 g$$

$$r_{\mathrm{outer}} = \left(\frac{3\times 1.26\ \mathrm{kg}}{4\pi 1\,000\ \mathrm{kg/m^3}}\right)^{1/3} = \boxed{6.70\ \mathrm{cm}}$$

(b) The mass of the ball is determined by the density of aluminum:

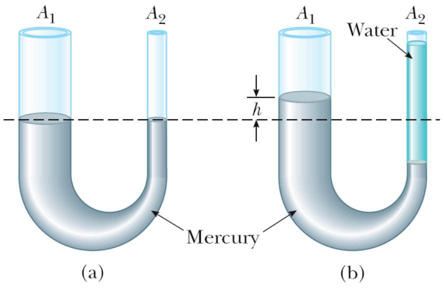
$$m = \rho_{\mathrm{Al}}V = \rho_{\mathrm{Al}}\left(\frac{4}{3}\pi r_0^3 - \frac{4}{3}\pi r_i^3\right)$$

$$1.26\ \mathrm{kg} = 2\,700\ \mathrm{kg/m^3}\left(\frac{4}{3}\pi\right)\left((0.067\ \mathrm{m})^3 - r_i^3\right)$$

$$1.11\times 10^{-4}\ \mathrm{m^3} = 3.01\times 10^{-4}\ \mathrm{m^3} - r_i^3$$

$$r_i = \left(1.89\times 10^{-4}\ \mathrm{m^3}\right)^{1/3} = \boxed{5.74\ \mathrm{cm}}$$

3. Mercury is poured into a U-tube as shown . The left arm of the tube has cross-sectional area A_1 of 10.0 cm² and the right arm has a cross-sectional area A_2 of 5.00 cm². One hundred grams of water are then poured into the right arm as shown in Figure P15.14b. (a) Determine the length of the water column in the right arm of the U-tube. (b) Given that the density of mercury is 13.6 g/cm³, what distance *h* does the mercury rise in the left arm?



(a) Using the definition of density, we have

$$h_w = \frac{m_{\mathrm{water}}}{A_2\rho_{\mathrm{water}}} = \frac{100\ \mathrm{g}}{5.00\ \mathrm{cm^2}\left(1.00\ \mathrm{g/cm^3}\right)} = \boxed{20.0\ \mathrm{cm}}$$

(b) Sketch at the right represents the situation after the water is added. A volume (A_2h_2) of mercury has been displaced by water in the right tube. The additional volume of mercury now in the left tube is A_1h. Since the total volume of mercury has not changed,

$A_2h_2 = A_1h$ or $h_2 = \frac{A_1}{A_2}h$ (1)

At the level of the mercury–water interface in the right tube, we may write the absolute pressure as:

$$P = P_0 + \rho_{\mathrm{water}}gh_w$$

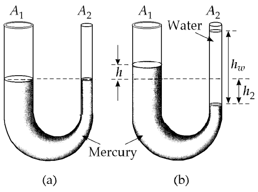
The pressure at this same level in the left tube is given by

$$P = P_0 + \rho_{\mathrm{Hg}}g(h + h_2) = P_0 + \rho_{\mathrm{water}}gh_w$$

which, using equation (1) above, reduces to

$$\rho_{\mathrm{Hg}}h\left[1+\frac{A_1}{A_2}\right] = \rho_{\mathrm{water}}h_w$$

Or $h = \frac{\rho_{\mathrm{water}}h_w}{\rho_{\mathrm{Hg}}\left(1+\frac{A_1}{A_2}\right)}$.

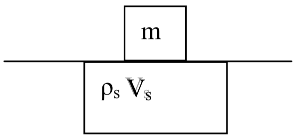


Thus, the level of mercury has risen a distance of

$$h = \frac{\left(1.00\ \mathrm{g/cm^3}\right)\left(20.0\ \mathrm{cm}\right)}{\left(13.6\ \mathrm{g/cm^3}\right)\left(1+\frac{10.0}{5.00}\right)} = \boxed{0.490\ \mathrm{cm}}$$ above the original level

4 A Styrofoam slab has thickness *h* and density ρ_s. When a swimmer of mass *m* is resting on it, the slab floats in fresh water with its top at the same level as the water surface. Find the area of the slab.

SOLUTION:
The weight of the slab and the swimmer are counterbalanced by the buoyancy of the slab (weight of the displaced water)



$$F_g = B$$

$$mg + \rho_s V_{slab} g = V_{slab} \rho_w g$$

$$mg = \rho_w V_{slab} g - V_s \rho_s g$$

$$m = (\rho_w - \rho_s) V_s$$

$$\frac{m}{(\rho_w - \rho_s)} = A_s h \qquad \boxed{\frac{m}{(\rho_w - \rho_s)h} = A_s}$$

5 A large storage tank, open at the top and filled with water, develops a small hole in its side at a point 16.0 m below the water level. If the rate of flow from the leak is 2.50 × 10^{-3} m^3/min, determine (a) the speed at which the water leaves the hole and (b) the diameter of the hole.

$P_1 = P_0$. Note $P_2 = P_0$.

Flow rate $= 2.50 \times 10^{-3}\ \text{m}^3/\text{min} = 4.17 \times 10^{-5}\ \text{m}^3/\text{s}$.

(a) $A_1 >> A_2$ so $v_1 << v_2$

Assuming $v_1 = 0$,

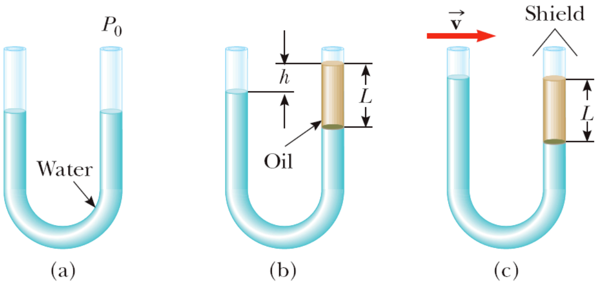
$$P_1 + \frac{\rho v_1^2}{2} + \rho g y_1 = P_2 + \frac{\rho v_2^2}{2} + \rho g y_2$$

$$v_2 = (2gy_1)^{1/2} = [2(9.80)(16.0)]^{1/2} = \boxed{17.7\ \text{m/s}}$$

(b) Flow rate $= A_2 v_2 = \left(\frac{\pi d^2}{4}\right)(17.7) = 4.17 \times 10^{-5}\ \text{m}^3/\text{s}$

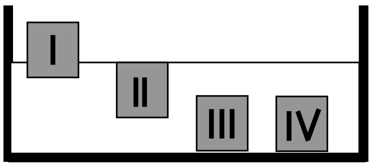
$$d = \boxed{1.73 \times 10^{-3}\ \text{m}} = 1.73\ \text{mm}$$

6 A U-tube open at both ends is partially filled with water (Fig. P15.61a). Oil having a density 750 kg/m³ is then poured into the right arm and forms a column *L* = 5.00 cm high (Fig. P15.61b). (a) Determine the difference *h* in the heights of the two liquid surfaces. (b) The right arm is then shielded from any air motion while air is blown across the top of the left arm until the surfaces of the two liquids are at the same height (Fig. P15.61c). Determine the speed of the air being blown across the left arm. Take the density of air as 1.29 kg/m³. CHECK PAGE 3



7 Four blocks of the same volume have different masses. The diagram shows the equilibrium position of the three of these blocks. Find the equilibrium position of the block III. In two/three clear sentences justify your answer

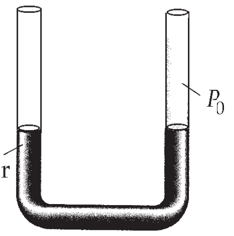
M(I) < M(II) < M(III) < M(IV)



Block III is heavier than block II. Since block II has the same density as fluid, the block III will sink (it s density is higher than the density of the fluid)

Variation of atmospheric pressure with altitude is included in this solution. Because of the small distances involved, this effect is unimportant in the final answers.

(a)



(a) Consider the pressure at points A and B in part (b) of the figure:

Using the left tube: $P_A = P_{\text{atm}} + \rho_a g h + \rho_w g(L-h)$ where the second term is due to the variation of air pressure with altitude.

Using the right tube: $P_B = P_{\text{atm}} + \rho_0 g L$

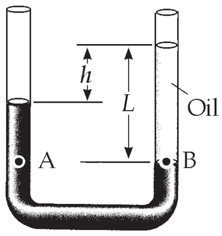
But Pascal's principle says that $P_A = P_B$.

Therefore, $P_{\text{atm}} + \rho_0 g L = P_{\text{atm}} + \rho_a g h + \rho_w g(L-h)$

or $(\rho_w - \rho_a)h = (\rho_w - \rho_0)L$, giving

$$h = \left(\frac{\rho_w - \rho_0}{\rho_w - \rho_a}\right)L = \left(\frac{1\,000-750}{1\,000-1.29}\right)5.00\text{ cm} = \boxed{1.25\text{ cm}}$$

(b)



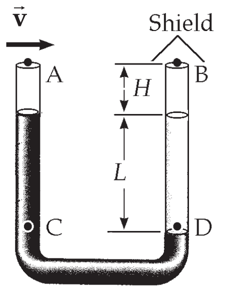
(b) Consider part (c) of the diagram showing the situation when the air flow over the left tube equalizes the fluid levels in the two tubes. First, apply Bernoulli's equation to points A and B $(y_A = y_B,\ v_A = v,\ \text{and } v_B = 0)$

This gives: $P_A + \frac{1}{2}\rho_a v^2 + \rho_a g y_A = P_B + \frac{1}{2}\rho_a (0)^2 + \rho_a g y_B$

and since $y_A = y_B$, this reduces to: $P_B - P_A = \frac{1}{2}\rho_a v^2$ (1)

Now consider points C and D, both at the level of the oil–water interface in the right tube. Using the variation of pressure with depth in static fluids, we have:

(c)



$P_C = P_A + \rho_a g H + \rho_w g L$ and $P_D = P_B + \rho_a g H + \rho_0 g L$

But Pascal's principle says that $P_C = P_D$. Equating these two gives:

$P_B + \rho_a g H + \rho_0 g L = P_A + \rho_a g H + \rho_w g L$ or $P_B - P_A = (\rho_w - \rho_0)gL$ (2)

Substitute equation (1) for $P_B - P_A$ into (2) to obtain $\frac{1}{2}\rho_a v^2 = (\rho_w - \rho_0)gL$

or $$v = \sqrt{\frac{2gL(\rho_w - \rho_0)}{\rho_a}} = \sqrt{2\left(9.80\text{ m/s}^2\right)(0.050\,0\text{ m})\left(\frac{1\,000-750}{1.29}\right)}$$

$$v = \boxed{13.8\text{ m/s}}$$

The fluid in the hydraulic jack is originally exerting the same pressure as the air outside. This pressure P_0 results in zero net force on either piston. For the equilibrium of piston 2 we require

$$500\text{ lb} = (P - P_0)A = (P - P_0)\pi\left(\frac{1.5\text{ in.}}{2}\right)^2$$

Let F_1 represent the force the lever bar exerts on piston 1. Then similarly

$$F_1 = (P - P_0)\pi\left(\frac{0.25\text{ in.}}{2}\right)^2$$

We ignore the weights of the pistons, sliding friction, and the slight difference in fluid pressure *P* due to the height difference between points 1 and 2. By division

$$\frac{F_1}{500\text{ lb}} = \left(\frac{0.25\text{ in.}}{1.5\text{ in.}}\right)^2$$

$$F_1 = \frac{500\text{ lb}}{36}$$

We say the hydraulic lift has an ideal mechanical advantage of 36. Next for the lever bar we ignore weight and friction, assume equilibrium, and take torques about the fixed hinge.

$$\sum \tau = 0 \qquad F_1(2\text{ in.}) - F(12\text{ in.}) = 0 \qquad F = \frac{F_1}{6}$$

The lever has an ideal mechanical advantage of 6. By substitution,

$$F = \frac{500\text{ lb}}{36 \cdot 6} = \boxed{2.31\text{ lb}}$$