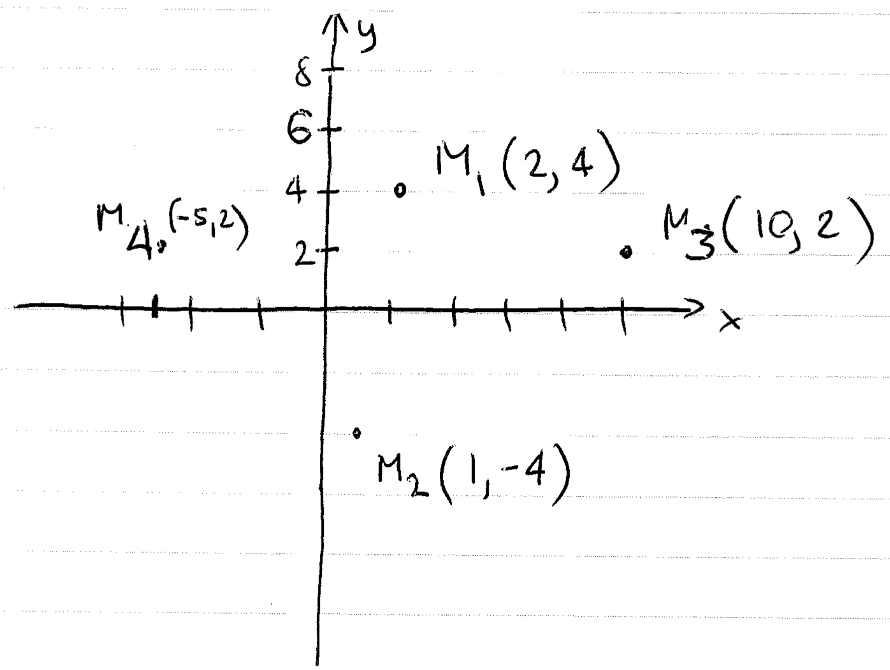
1. $\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{\frac{2\pi rad}{1s} - 0}{30s} = \frac{2\pi}{30} \frac{rad}{s^2}$

$\alpha = \frac{\pi}{15} rad/s^2$

2.



$M_1 = 3kg$
$M_2 = 2kg$
$M_3 = 1kg$
$M_4 = 5kg$

a) $I_x = M_1 y_1^2 + M_2 y_2^2 + M_3 y_3^2 + M_4 y_4^2 =$

$I_x = 3 \cdot (4)^2 + 2(-4)^2 + 1 \cdot (2)^2 + 5 (2)^2 = 48 + 32 + 4 + 20$

$I_x = 104 \ kg \ m^2$

b) $I_y = M_1 x_1^2 + M_2 x_2^2 + M_3 x_3^2 + M_4 x_4^2$

$I_y = 3(2)^2 + 2(1)^2 + 1(10)^2 + 5(-5)^2 = 12 + 2 + 100 + 125$

$I_y = 239 \ kgm^2$

c) $K_x = \frac{1}{2} I_x \omega^2 = \frac{1}{2} 104 \cdot 4^2 \, J = 832 \, J$

d) $K_y = \frac{1}{2} I_y \omega^2 = \frac{1}{2} 239 \cdot 4^2 \, J = 1912 \, J$

3

full sphere

$M = 10\,kg$

$R = 0.2\,m$

$\rho = \frac{M}{\frac{4}{3}\pi R^3} = 298.4\,kg/m^3$ ✓

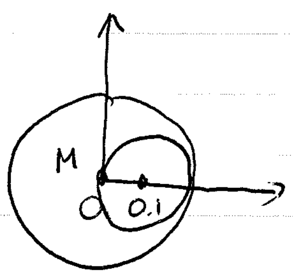
small sphere of negative mass

$r = 10\,cm = 0.1\,m$

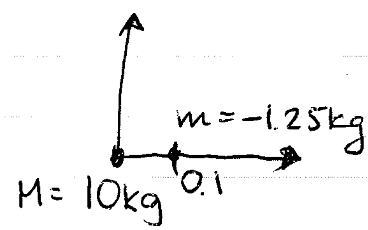
$\rho = 298.4\,kg/m^3$

$|m| = \rho \cdot \frac{4}{3}\pi R^3$

$|m| = 1.25\,kg \Rightarrow m = -1.25\,kg$ ✓



$\equiv$



$$X_{CM} = \frac{M \cdot 0 + m(0.1)}{M+m} = \frac{-1.25(0.1)}{10-1.25}$$

$$X_{CM} = \frac{-1.25}{8.75} = -\frac{5}{35} = -\frac{1}{7}\,m$$

(B) $I = I_M + I_m$

$$I_M = I_{CM_{SPHERE}} + MD^2 = \frac{2}{5}MR^2 + M\left(+\frac{1}{7}\right)^2$$

$$= \frac{2}{5}\left(\frac{2}{10}\right)^2 \cdot 10 + 10\,\frac{1}{49} = \frac{8}{50} + \frac{10}{49} = 1.964$$

$$I_m = -\left(I_{CM_{SPHERE}} + mD^2\right) = -\left(\frac{2}{5}mr^2 + m\left(\frac{1}{7}+\frac{1}{10}\right)^2\right)$$

$$I_m = -\left[\frac{2}{5}\,1.25(0.1)^2 + 1.25\left(\frac{1}{7}+\frac{1}{10}\right)^2\right]$$

$$I_m = -\left(\frac{0.025}{5} + \frac{1.25 \cdot 17^2}{4900}\right) = -0.0787$$

$$I_{Tot} = 1.964 - 0.0787 = \underline{1.885 \text{ kg m}^2}$$

4

$$E_f = E_i$$

$$\frac{1}{2} I \omega^2 + \frac{1}{2} m v^2 = m g h$$

$$\omega = \frac{v}{r}$$

$$\frac{1}{2} I \left(\frac{v}{r}\right)^2 + \frac{1}{2} m v^2 = m g h$$

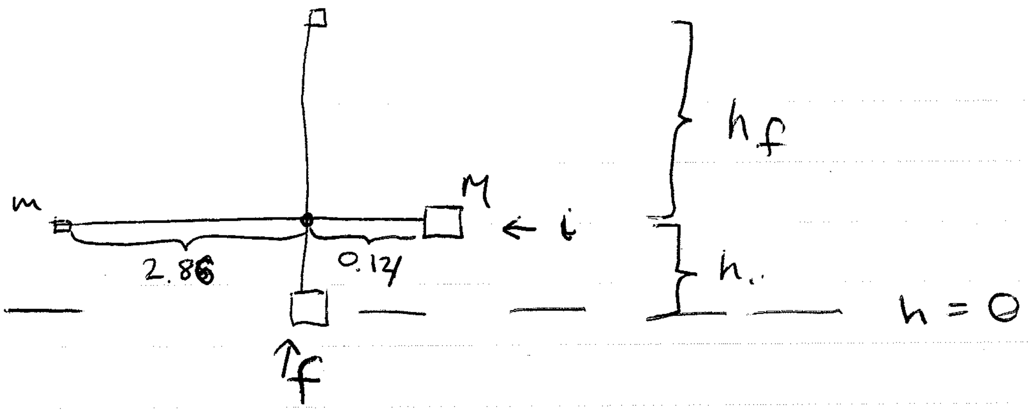
$$\frac{I v^2}{r} = 2 m g h + - m v^2$$

$$I = \frac{2 m g h - m v^2}{v^2} r$$

$$I = m r^2 \left(\frac{2 g h - v^2}{v^2}\right)$$

$$I = m r^2 \left(\frac{2 g h}{v^2} - 1\right)$$

5



$$E_i = Mgh_i + mgh_i$$

$$E_f = mg(h_i + h_f) + \frac{1}{2} m v_1^2 + \frac{1}{2} M v_2^2$$

this is rigid body that rotates $\longrightarrow$ $\frac{v_1}{h_i} = \frac{v_2}{h_i} = \omega_f$

$E_i = E_f$ — system conserves mech. energy

$$Mgh_i + \cancel{mgh_i} = \cancel{mgh_i} + mgh_f + \frac{1}{2} m v_1^2 + \frac{1}{2} M v_2^2$$

$$Mgh_i = mgh_f + \frac{1}{2} m v_1^2 + \frac{1}{2} M \left(\frac{h_i}{h_f}\right)^2 v_1^2$$

$$Mgh_i - mgh_f = \left[\frac{1}{2} m + \frac{1}{2} M \left(\frac{h_i}{h_f}\right)^2\right] v_1^2$$

$$(60)(9.8)(0.14) - (0.12)(9.8)(2.86) = \left(0.07 + 30\left(\frac{0.14}{2.86}\right)^2\right) v_1^2$$

$$82.32 - 3.36 = (0.142)\, v_1^2$$

$$78.96 = 0.142\, v_1^2$$

$$556.5 = v_1^2$$

$$v = \sqrt{556.5} = 23.6 \text{ m/s}$$

$$(84.93 \text{ km/h})$$