PHY 1321 PHY1331
Principles of Physics I
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Assignment 1
Released: Fri Sept 14th Due Friday 21st 10 AM

1 By algebraic manipulation of the first two kinematic equations for one-dimensional motion:

$$1)v_f = v_i + at \qquad 2)x_f = x_i + v_i t + \tfrac{1}{2}at^2$$

Obtain the other two kinematic equations: $3)v_f^2 - v_i^2 = 2a\Delta x \qquad 4)x_f = x_i + \tfrac{1}{2}(v_i + v_f)t$

SOLUTION:

$$1)v_f = v_i + at \Rightarrow t = \frac{(v_f - v_i)}{a}$$

$$2)x_f = x_i + v_i\frac{(v_f - v_i)}{a} + \tfrac{1}{2}a\frac{(v_f - v_i)^2}{a^2} \Rightarrow x_f - x_i = v_i\frac{(v_f - v_i)}{a} + \tfrac{1}{2a}\left(v_f^2 - 2v_iv_f + v_i^2\right) \Rightarrow$$

$$\Rightarrow 2a(x_f - x_i) = 2v_i(v_f - v_i) + \left(v_f^2 - 2v_iv_f + v_i^2\right) \Rightarrow 2a(x_f - x_i) = 2v_iv_f - 2v_iv_i + v_f^2 - 2v_iv_f + v_i^2 \Rightarrow 2a(x_f - x_i) = v_f^2 - v_i^2$$

$$1)v_f = v_i + at \Rightarrow t = \frac{(v_f - v_i)}{a}$$

$$2)x_f = x_i + v_i t + \tfrac{1}{2}a\frac{(v_f - v_i)}{a}t \Rightarrow x_f = x_i + v_i t + \tfrac{1}{2}(v_f - v_i)t \Rightarrow x_f = x_i + v_i t + \tfrac{1}{2}v_f t - \tfrac{1}{2}v_i t \Rightarrow x_f = x_i + \tfrac{1}{2}(v_f + v_i)t$$

2. The driver of a car slams on the brakes when he sees a tree blocking the road. The car slows uniformly with an acceleration of –5.60 m/s2 for 4.20 s, making straight skid marks 62.4 m long ending at the tree. With what speed does the car then strike the tree?

SOLUTION:
In the simultaneous equations:

$$\left\{\begin{matrix} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t \end{matrix}\right\} \text{ we have } \left\{\begin{matrix} v_{xf} = v_{xi} - \left(5.60\ \text{m/s}^2\right)(4.20\ \text{s}) \\ 62.4\ \text{m} = \frac{1}{2}\left(v_{xi} + v_{xf}\right)(4.20\ \text{s}) \end{matrix}\right\}.$$

So substituting for v_{xi} gives $62.4\ \text{m} = \frac{1}{2}\left[v_{xf} + \left(56.0\ \text{m/s}^2\right)(4.20\ \text{s}) + v_{xf}\right](4.20\ \text{s})$

$14.9\ \text{m/s} = v_{xf} + \frac{1}{2}\left(5.60\ \text{m/s}^2\right)(4.20\ \text{s})$. Thus $v_{xf} = \boxed{3.10\ \text{m/s}}$.

3 The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. After 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

SOLUTION: $y = 3.00t^3$: At $t = 2.00\ \text{s}$, $y = 3.00(2.00)^3 = 24.0\ \text{m}$ and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0\ \text{m/s} \uparrow.$$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2}gt^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)t^2.$$

Setting $y_b = 0$, $0 = 24.0 + 36.0t - 4.90t^2$.

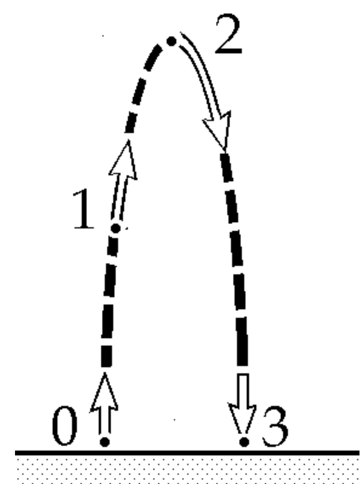
Solving for t, (only positive values of t count), $\boxed{t = 7.96\ \text{s}}$.

Assignment 1 Cont.

4 A rocket is fired vertically upward from a well. A catapult gives it initial velocity 80.0 m/s at ground level. Its engines then fire and it accelerates upward at 4.00 m/s^2 until it reaches an altitude of 1 000 m. At that point its engines fail and the rocket goes into free fall, with an acceleration of −9.80 m/s^2. (a) How long is the rocket in motion above the ground? (b) What is its maximum altitude? (c) What is its velocity just before it collides with the Earth? (You will need to consider the motion while the engine is operating separate from the free-fall motion

2 SOLUTION:

Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact. Below are the data found for each phase of the rocket's motion.



(0 to 1) $v_f^2 - (80.0)^2 = 2(4.00)(1\,000)$ so $v_f = 120$ m/s

$120 = 80.0 + (4.00)t$ giving $t = 10.0$ s

(1 to 2) $0 - (120)^2 = 2(-9.80)(x_f - x_i)$ giving $x_f - x_i = 735$ m

$0 - 120 = -9.80t$ giving $t = 12.2$ s

This is the time of maximum height of the rocket.

(2 to 3) $v_f^2 - 0 = 2(-9.80)(-1\,735)$

$v_f = -184 = (-9.80)t$ giving $t = 18.8$ s

(a) $t_{\text{total}} = 10 + 12.2 + 18.8 = \boxed{41.0 \text{ s}}$

(b) $(x_f - x_i)_{\text{total}} = \boxed{1.73 \text{ km}}$

(c) $v_{\text{final}} = \boxed{-184 \text{ m/s}}$

		t	x	v	a
0	Launch	0.0	0	80	+4.00
#1	End Thrust	10.0	1 000	120	+4.00
#2	Rise Upwards	22.2	1 735	0	-9.80
#3	Fall to Earth	41.0	0	-184	-9.80

5 A rock is dropped from rest into a well. (a) The sound of the splash is heard 2.40 s after the rock is released from rest. How far below the top of the well is the surface of the water? The speed of sound in air (at the ambient temperature) is 336 m/s. (b) **What If?** If the travel time for the sound is neglected, what percentage error is introduced when the depth of the well is calculated?

SOLUTION:

(a) $d = \frac{1}{2}(9.80)t_1^2$ $d = 336t_2$

$t_1 + t_2 = 2.40$ $336t_2 = 4.90(2.40 - t_2)^2$

$4.90t_2^2 - 359.5t_2 + 28.22 = 0$ $t_2 = \dfrac{359.5 \pm \sqrt{359.5^2 - 4(4.90)(28.22)}}{9.80}$

$t_2 = \dfrac{359.5 \pm 358.75}{9.80} = 0.076\,5$ s so $d = 336t_2 = \boxed{26.4 \text{ m}}$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2$ m, an error of. $\boxed{6.82\%}$