.MATH 1300 A & B-MIDTERM # 1-2014.

NAME and I.D.# version 1

Instructions: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 40 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. **You may use the backs of pages.**

For long answer questions, YOU MUST SHOW YOUR WORK.

NO CALCULATORS. NO BOOKS. NO NOTES.

If you need additional scrap paper, it will be provided by the proctors.

Multiple Choice Answers:

Question	1	2	3	4
Answer	D	C	C	B

Multiple Choice Questions (1-4)

Question 1 *Solve the equation* $\log_2(x+1) + \log_2(x-2) = 2$ *for* x.

A) $x = 2$ **B)** $x = -2$ **C)** $x = 3$ and $x = -2$ **D)** $x = 3$ **E)** $x = -3$ and $x = 1$

$$\log_2 (x+1)(x-2) = 2$$

$$x^2 - x - 2 = 4$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3, \quad x = -2 \ X$$

Question 2 *Find the value or values of* x *for which the following function has slope 0.*

$$f(x) = xe^{-\frac{x}{2}}$$

A) $x = 0$ **B)** $x = 1$ **C)** $x = 2$ **D)** $x = 4$ **E)** $x = \frac{1}{2}$

$$f'(x) = e^{-\frac{x}{2}} - \frac{1}{2} x e^{-x/2}$$

$$= e^{-x/2}\left(1 - \tfrac{1}{2}x\right) = 0$$

$$x = 2$$

Question 3 *Calculate*

$$\lim_{x\to 2} \frac{x^2-3x+2}{x^2-4}$$

A) 0 **B)** $\frac{1}{2}$ **C)** $\frac{1}{4}$ **D)** 1 **E)** The limit does not exist.

$x \neq 2$

$$\lim_{x\to 2} \frac{(x-1)(x-2)}{(x-2)(x+2)} = \lim_{x\to 2} \frac{(x-1)}{x+2} = 1/4$$

Question 4 *Find the value of a, for which the following function is continuous everywhere:*

$$f(x) = \begin{cases} |x-3|-1 & \text{if } x < -1 \\ ax+2 & \text{if } x \geq -1 \end{cases}$$

A) 1 **B)** -1 **C)** $-\frac{2}{3}$ **D)** $\frac{1}{2}$ **E)** 0

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$$

$$|-1-3|-1 = -a+2$$

$$3 = -a+2 \qquad a = -1$$

Long Answer Questions (5-7)

Question 5 (8 points)

Suppose a function $y = f(x)$ is defined implicitly by the equation

$$x^2y + 2x - y^2 = 7$$

5/ • *Find* $\frac{dy}{dx}$.

3/ • *Find the equation of a tangent line to the curve at the point* $(2, 3)$.

$2xy + x^2y' + 2 - 2yy' = 0$ 3 points

$y'(x^2 - 2y) = -2xy - 2$

$y' = \frac{-2xy - 2}{x^2 - 2y}$ ← 2points.

$y'|_{(2,3)} = \frac{-12 - 2}{4 - 6} = \frac{-14}{-2} = 7$ ← 1point

$y - 3 = 7(x - 2)$

$y = 7x - 11$ ← 2points

Question 6 (6 points)

Using only the definition of derivative as a limit, calculate $f'(x)$ where

$$f(x) = \sqrt{x-2}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x-2} - \sqrt{x-2}}{\Delta x}$$ 1 point

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x-2} - \sqrt{x-2}}{\Delta x} \times \frac{\sqrt{x+\Delta x-2} + \sqrt{x-2}}{\sqrt{x+\Delta x-2} + \sqrt{x-2}}$$ ← 2 points

$$= \lim_{\Delta x \to 0} \frac{x+\Delta x-2-(x-2)}{\Delta x\left(\sqrt{x+\Delta x-2} + \sqrt{x-2}\right)} =$$ 1 point

$$\lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x-2} + \sqrt{x-2}\right)} = \frac{1}{2\sqrt{x-2}}$$ 2 points.

Question 7 (6 points)

One thousand dollars is invested at a rate of 5%,

2/ • *How much will be in the account in 6 years if compounded 12 times per year?*

4/ • *How long is required for the initial investment to triple if the account compounds continuously?*

You do not need to simplify your answers.

$A(t) = P_0\left(1+\frac{r}{n}\right)^{nt}$ or 72 ← 2 point

$= 10^3\left(1+\frac{0.05}{12}\right)^{12(6)}$

$A(t) = P_0 e^{rt}$ ← 2 points

$3P_0 = P_0 e^{0.05t}$

$3 = e^{0.05t}$ ← 1

$\ln 3 = 0.05t$

$t = \frac{\ln 3}{0.05}$ ←

1 points

Space for additional work

.MATH 1300 A & B-MIDTERM # 1-2014

NAME and I.D.# version 2

Instructions: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 40 points.

Place your answers to the multiple choice questions in the boxes below. All your work on the long answer questions must be clearly marked. **You may use the backs of pages.**

For long answer questions, YOU MUST SHOW YOUR WORK.

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If you need additional scrap paper, it will be provided by the proctors.

MULTIPLE CHOICE ANSWERS:

Question	1	2	3	4
Answer	A	A	E	B

Multiple Choice Questions (1-4)

Question 1 *Solve the equation* $\log_6(x-3)+\log_6(x+2)=1$ *for* x.

✓ **A)** $x=4$ **B)** $x=-4$ **C)** $x=4$ and $x=-2$ **D)** $x=2$ **E)** $x=-3$ and $x=4$

$$\log_6(x-3)(x+2)=1$$

$$x^2-x-6=6$$

$$x^2-x-12=0$$

$$(x-4)(x+3)=0$$

$x=4$ ✓

$x=-3$ ✗

Question 2 *Find all values of* x *for which the following function has slope 0.*

$$f(x)=x^2e^{2x}$$

✓ **A)** $x=0$ and $x=-1$ **B)** $x=1$ and $x=2$ **C)** $x=2$ **D)** $x=4$ **E)** $x=\frac{1}{2}$

$$f'(x)=2xe^{2x}+2x^2e^{2x}$$

$$=xe^{2x}(2+2x)=0$$

$x=-1$ and $x=0$

Question 3 *Calculate*

$$\lim_{x\to -3} \frac{x^2+x-6}{x^2+2x-3}$$

A) 0 **B)** $\frac{1}{4}$ **C)** The limit does not exist. **D)** 1 **E)** $\frac{5}{4}$

$$\lim_{x\to -3} \frac{(x+3)(x-2)}{(x+3)(x-1)} \overset{x\neq -3}{=} \lim_{x\to -3} \frac{(x-2)}{(x-1)}$$

$$= \frac{-5}{-4} = 5/4$$

Question 4 *Find the value of a, for which the following function is continuous everywhere:*

$$f(x) = \begin{cases} |x-2|+3 & \text{if } x < -1 \\ ax+2 & \text{if } x \geq -1 \end{cases}$$

A) 1 **B)** -4 **C)** $-\frac{2}{3}$ **D)** $\frac{1}{8}$ **E)** 4

$$\lim_{x\to -1^-} f(x) = \lim_{x\to 1^+} f(x)$$

$$|-1-2|+3 = -a+2$$

$$6 = -a+2$$

$$a = -4$$

Long Answer Questions (5-7)

Question 5 (8 points)

Suppose a function $y = f(x)$ is defined implicitly by the equation

$$x^2y + 2y - 2y^2 = -9$$

5/

- *Find* $\frac{dy}{dx}$.

1/3

- *Find the equation of a tangent line to the curve at the point* $(1, 3)$.

$$2xy + x^2y' + 2y' - 4yy' = 0$$

$$y'(x^2 + 2 - 4y) = -2xy$$

$$y' = \frac{-2xy}{x^2 + 2 - 4y}$$

$$y'\Big|_{(1,3)} = \frac{-6}{1+2-12} = \frac{-6}{-9} = 2/3$$

$$y - 3 = 2/3(x - 1)$$

$$y = 2/3x + 7/3$$

Question 6 (6 points)

Using only the definition of derivative as a limit, calculate $f'(x)$ where

$$f(x) = \sqrt{x+1}$$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x+1} - \sqrt{x+1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x+1} - \sqrt{x+1}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x+1} + \sqrt{x+1}}{\sqrt{x+\Delta x+1} + \sqrt{x+1}}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x+1} + \sqrt{x+1}\right)} = \frac{1}{2\sqrt{x+1}}$$

Question 7 (6 points)

One thousand dollars is invested at a rate of 4%,

2/4

- *How much will be in the account in 7 years if compounded 10 times per year?*
- *How long is required for the initial investment to double if the account compounds continuously?*

You do not need to simplify your answers.

$$A(t) = P_0\left(1+\frac{r}{n}\right)^{nt}$$

$$= 10^3\left(1+\frac{0.04}{10}\right)^{7(10)}$$

$$A(t) = P_0 e^{rt}$$

$$2P_0 = P_0 e^{0.04t}$$

$$2 = e^{0.04t} \qquad \ln 2 = 0.04t$$

$$t = \frac{\ln 2}{0.04}$$

Space for additional work

MATH 1300 A & B-MIDTERM # 1-2014.

NAME and I.D.# Version3

Instructions: This midterm exam consists of 4 multiple choice questions and 3 long answer questions. The multiple choice questions are worth 5 points each, and the long answer questions are as indicated. The total value of the exam is 40 points.

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For long answer questions, YOU MUST SHOW YOUR WORK.

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If you need additional scrap paper, it will be provided by the proctors.

MULTIPLE CHOICE ANSWERS:

Question	1	2	3	4
Answer	E	B.	E	B

Multiple Choice Questions (1-4)

Question 1 *Find all values of x for which the following function has slope 0.*

$$f(x) = x^2 e^{2x}$$

A) $x = \frac{1}{2}$ **B)** $x = 2$ **C)** $x = 1$ and $x = 2$ **D)** $x = 4$ **E)** $x = 0$ and $x = -1$

$f'(x) = 2xe^{2x} + x^2 2e^{2x} = 0$

$2xe^{2x}(1 + x) = 0$ $\quad x = 0$

$x = -1$

Question 2 *Calculate*

$$\lim_{x \to -3} \frac{x^2 + x - 6}{x^2 + 2x - 3}$$

A) 1 **B)** $\frac{5}{4}$ **C)** The limit does not exist. **D)** 0 **E)** $\frac{1}{4}$

$x \neq +3$

$$\lim_{x \to -3} \frac{(x+3)(x-2)}{(x+3)(x-1)} = \lim_{x \to -3} \frac{x-2}{x-1}$$

$$= \frac{-5}{-4} = \frac{5}{4}$$

Question 3 *Find the value of a, for which the following function is continuous everywhere:*

$$f(x) = \begin{cases} |x-2|+3 & \text{if } x < -1 \\ ax+2 & \text{if } x \geq -1 \end{cases}$$

A) 1 **B)** 4 **C)** $\frac{1}{8}$ **D)** $-\frac{2}{3}$ **E)** -4

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x)$

$|-1-2|+3 = -a+2$

$6 = -a+2 \quad a = -4$

Question 4 *Solve the equation* $\log_6(x-3) + \log_6(x+2) = 1$ *for* x.

A) $x = -4$ **B)** $x = 4$ **C)** $x = 4$ and $x = -3$ **D)** $x = 2$ **E)** $x = -2$ and $x = 4$

$\log_6(x-3)(x+2) = 1$

$x^2 - x - 6 = 6$

$x^2 - x - 12 = 0$

$(x+3)(x-4) = 0$

$x = 4$ ✓

$x = -3$ ✗

Long Answer Questions (5-7)

Question 5 (8 points)

Suppose a function $y = f(x)$ is defined implicitly by the equation

$$x^2y + 2y - 2y^2 = 4$$

5/

- *Find* $\frac{dy}{dx}$.

3/

- *Find the equation of a tangent line to the curve at the point* $(2, 2)$.

$$2xy + x^2y' + 2y' - 4yy' = 0$$

$$y'(x^2 + 2 - 4y) = -2xy$$

$$y' = \frac{-2xy}{x^2 + 2 - 4y}$$

$$y'|_{(2,2)} = \frac{-8}{4 + 2 - 8} = \frac{-8}{-2} = 4$$

$$y - 2 = 4(x - 2)$$

$$y = 4x - 6$$

Question 6 (6 points)

Using only the definition of derivative as a limit, calculate $f'(x)$ where

$$f(x) = \sqrt{x+3}$$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x+3} - \sqrt{x+3}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x+3} - \sqrt{x+3}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x+3} + \sqrt{x+3}}{\sqrt{x+\Delta x+3} + \sqrt{x+3}}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x+3} + \sqrt{x+3}\right)} = \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x+3} + \sqrt{x+3}}$$

$$= \frac{1}{2\sqrt{x+3}}$$

Question 7 (6 points)

One thousand dollars is invested at a rate of 3%,

- *How much will be in the account in 5 years if compounded 6 times per year?*
- *How long is required for the initial investment to double if the account compounds continuously?*

You do not need to simplify your answers.

$$A(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

$$= 10^3\left(1+\frac{0.03}{6}\right)^{30}$$

$$A(t) = Pe^{rt}$$

$$2P = Pe^{rt}$$

$$2 = e^{0.03t}$$

$$\ln 2 = 0.03t$$

$$t = \frac{\ln 2}{0.03}$$

Space for additional work