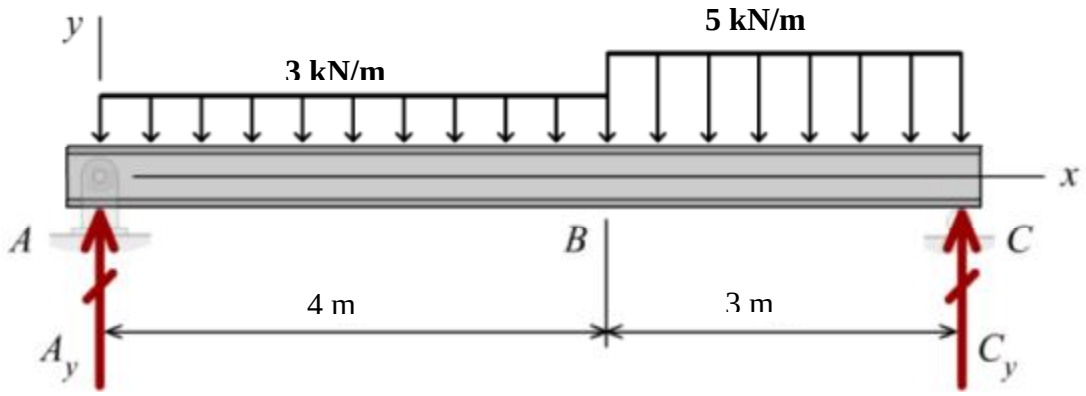
## SOLUTION

Problem 1)

a) The equations for shear force V and bending moment M




$\Sigma M_C = (3\ kN/m)(4\ m)(5\ m) + (5\ kN/m)(3\ m)(1.5\ m) - (A_y)(7\ m) = 0$

$\therefore A_y = 11.8\ kN \uparrow$

$\Sigma F_y = A_y + C_y - (3\ kN/m)(4\ m) - (5\ kN/m)(3\ m) = 11.8\ kN + C_y - 12\ kN - 15\ kN = 0$

$\therefore C_y = 15.2\ kN \uparrow$

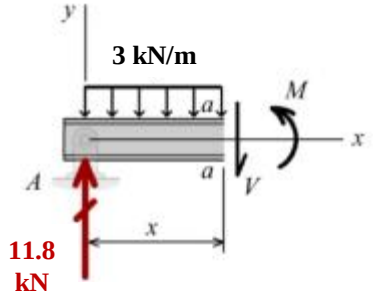
**Section *a-a*:** (The interval $0 \leq x < 4$ m)

$\Sigma F_y = 11.8 - (3)(x) - V = 0$

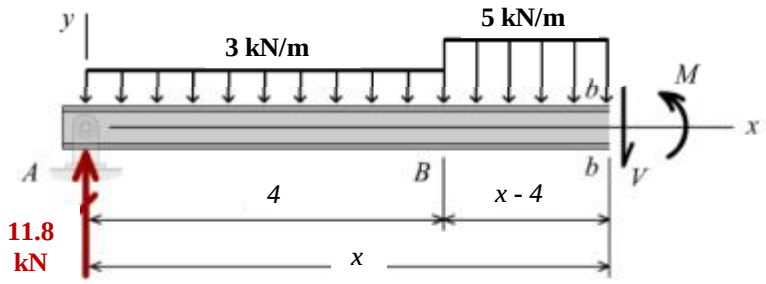
$\therefore \boxed{V = -3x + 11.8\ kN}$

$\Sigma M_{a-a} = -(11.8)(x) + (3)(x)(x/2) + M = 0$

$\therefore \boxed{M = -1.5x^2 + 11.8x\ kN.m}$



**Section *b-b*:** (The interval 4 m ≤ x < 7 m)



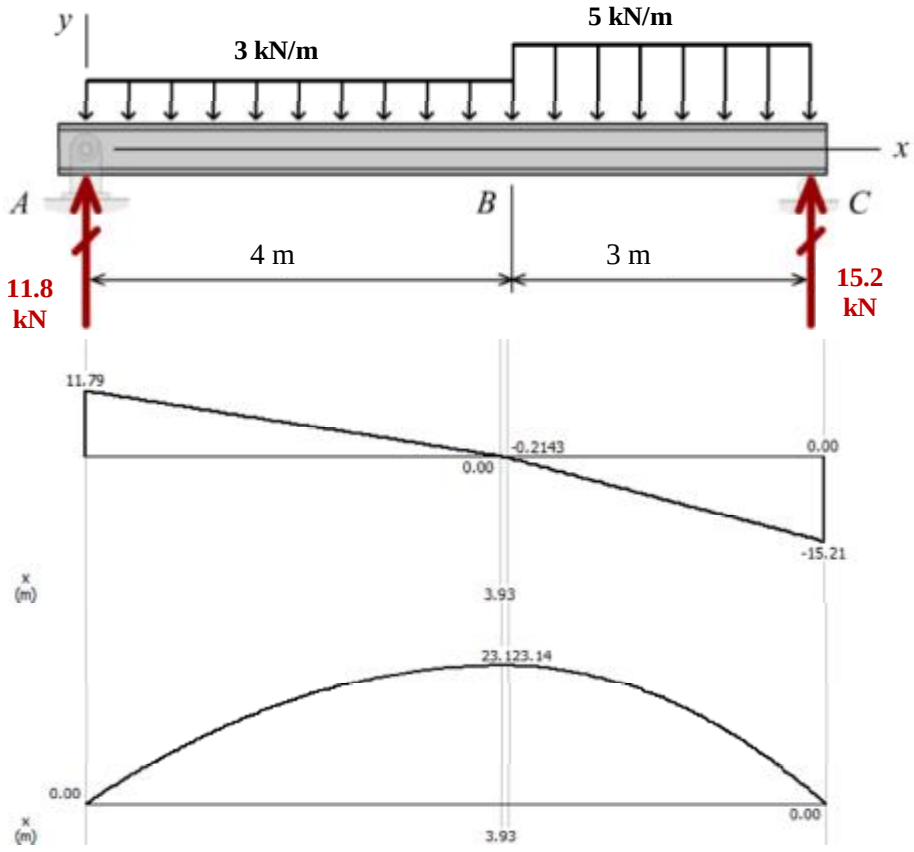

$\Sigma F_y = 11.8 - (3)(4) - (5)(x - 4) - V = 0$

$\therefore$ $V = -5x + 19.8$ kN

$\Sigma M_{b\text{-}b} = -(11.8)(x) + (3)(4)(x - 2) + (5)(x - 4)(x - 4)/2 + M = 0$

$\therefore$ $M = -2.5x^2 + 19.8x - 16$ kN.m

b) The shear force and bending moment diagrams

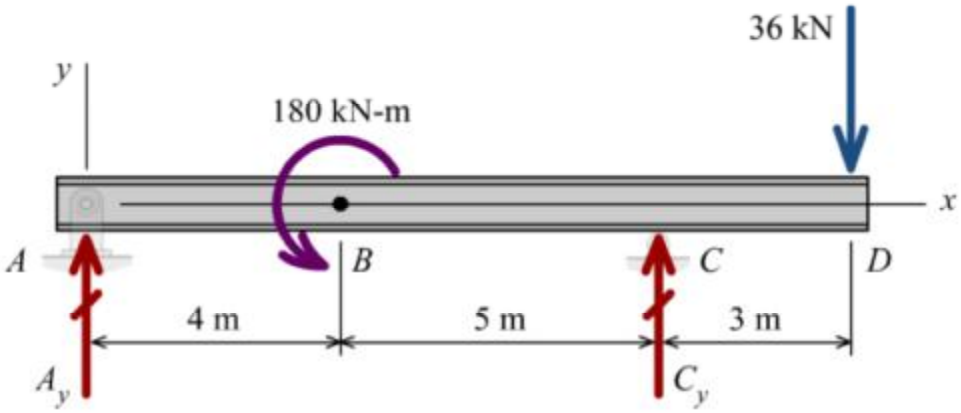



**V** (kN)

**M** (kN.m)

Problem 2)

a) The equations for shear force V and bending moment M



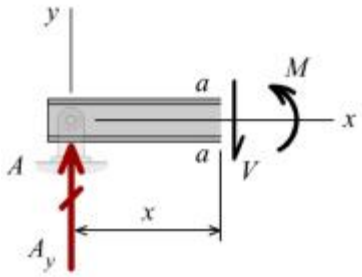

$\Sigma M_A = 180 \text{ kN.m} + (C_y)(9 \text{ m}) - (36 \text{ kN})(12 \text{ m}) = 0$

$\therefore C_y = 28 \text{ kN} \uparrow$

$\Sigma F_y = A_y + C_y - 36 \text{ kN} = A_y + 28 \text{ kN} - 36 \text{ kN} = 0$

$\therefore A_y = 8 \text{ kN} \uparrow$

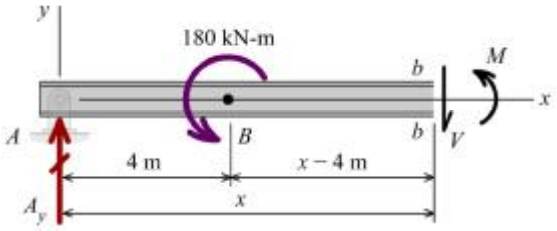
**Section *a-a*:** (The interval $0 \le x < 4$ m)




$\Sigma F_y = A_y - V = 8 - V = 0$ $\quad \therefore$ V = 8 kN

$\Sigma M_{a\text{-}a} = -(A_y)(x) + M = -(8)(x) + M = 0$ $\quad \therefore$ M = 8 x kN

**Section *b-b*:** (The interval $4 \text{ m} \le x < 9$ m)




$\Sigma F_y = A_y - V = 8 - V = 0$ $\quad \therefore$ V = 8 kN

$\Sigma M_{b\text{-}b} = -(A_y)(x) + 180 + M$

$= -(8)(x) + 180 + M = 0$ $\quad \therefore$ M = 8 x – 180 kN.m

**Section *c-c*:** (The interval 9 m ≤ x < 12 m)

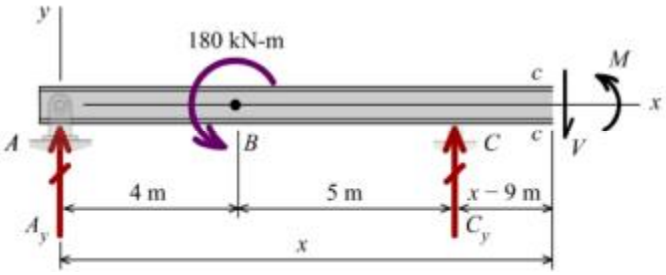
$\Sigma F_y = A_y + C_y - V = 8 + 28 - V = 0$

$\therefore$ $\boxed{V = 36 \text{ kN}}$

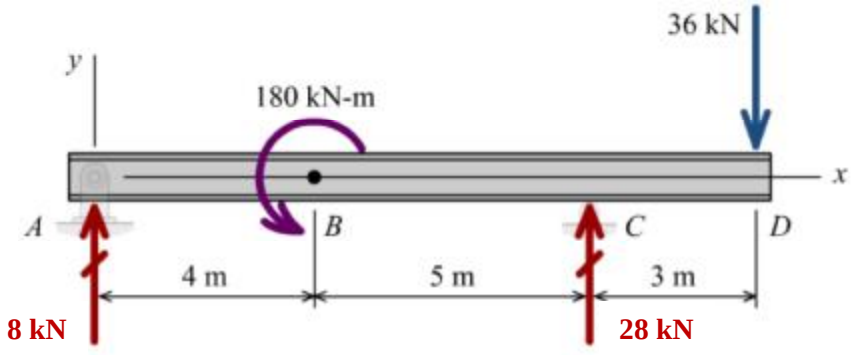
$\Sigma M_{c\text{-}c} = -(A_y)(x) - (C_y)(x - 9) + 180 + M$

$= -(8)(x) - (28)(x - 9) + 180 + M = 0$

$\therefore$ $\boxed{M = 36x - 432 \text{ kN.m}}$

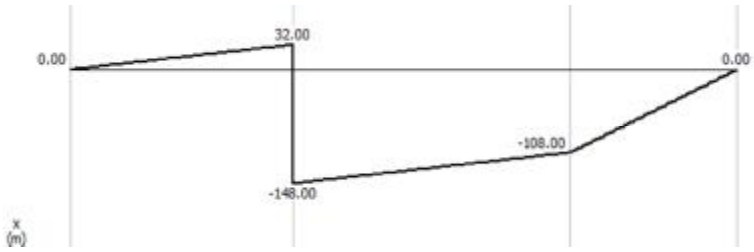



b) The shear force and bending moment diagrams




**V** (kN)

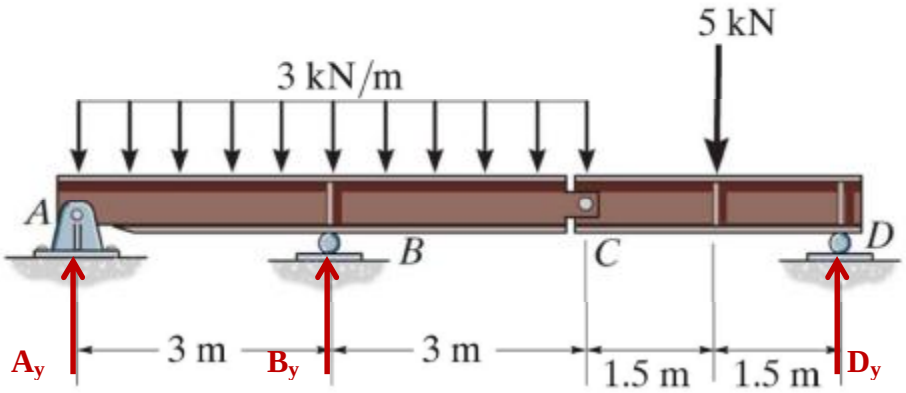

**M** (kN.m)




c) Maximum positive and negative bending moments and their respective locations

They are shown on the diagrams.

Problem 3)



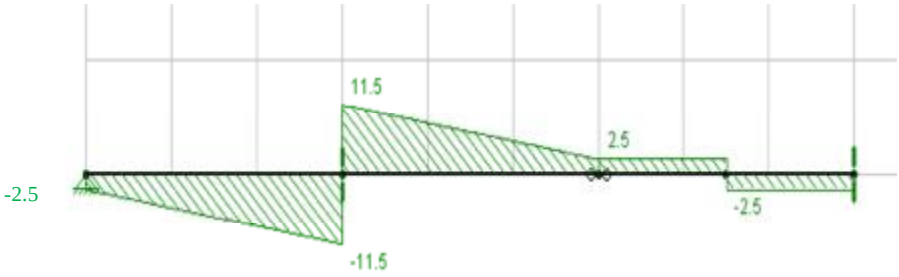

By establishing equilibrium on segment *CD* (moments are assumed positive counterclockwise):

$$\left.\begin{aligned}\sum F_y &= C_y + D_y - 5 = 0\\ \sum M_C &= -(5\times 1.5) + (D_y \times 3) = 0\end{aligned}\right\} \Rightarrow \begin{aligned} C_y &= 2.5 \text{ kN}\\ D_y &= 2.5 \text{ kN}\end{aligned}$$

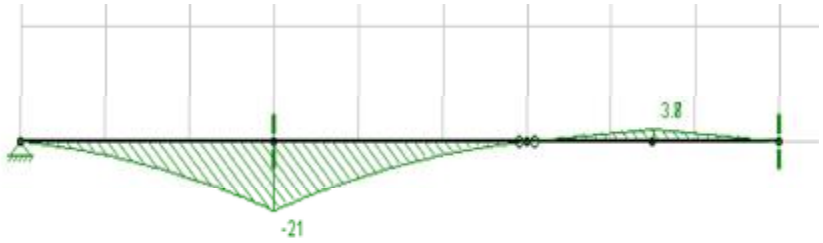
Likewise, by establishing equilibrium on segment *ABC*:

$$\left.\begin{aligned}\sum F_y &= A_y + B_y - (3\times 6) - C_y = 0\\ \sum M_A &= -(3\times 6\times 3) + (B_y \times 3) - (C_y \times 6) = 0\end{aligned}\right\} \Rightarrow \begin{aligned} A_y &= -2.5 \text{ kN}\\ B_y &= 23 \text{ kN}\end{aligned}$$

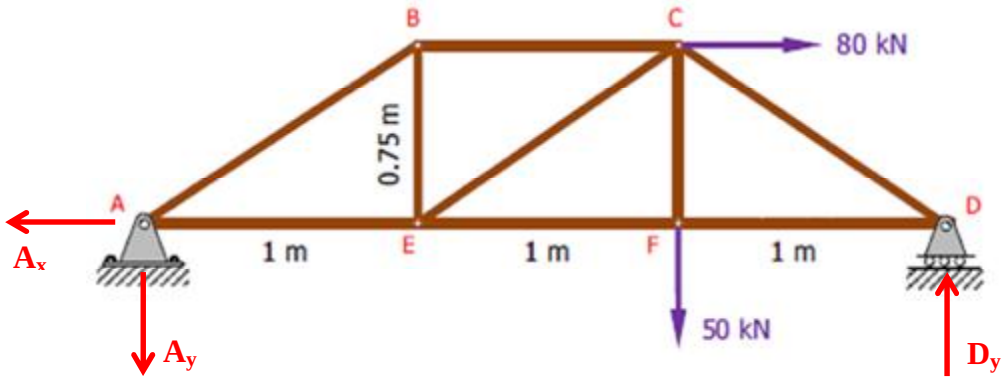
**The shear force diagram for the compound beam is shown below:** (Units in kN)




**The bending moment diagram for the compound beam is shown below:** (Units in kN.m)



Problem 4)




**(1) External reactions**

$\Sigma M_D = (A_y)(3) + (50)(1) - (80)(0.75) = 0$

$A_y = 3.33$ kN

$\Sigma F_x = 80 - A_x = 0$

$A_x = 80$ kN

$\Sigma F_y = D_y - A_y - 50 = D_y - 3.33 - 50 = 0$

$D_y = 53.33$ kN

**(2) Forces in truss members**

**<u>At joint A</u>**

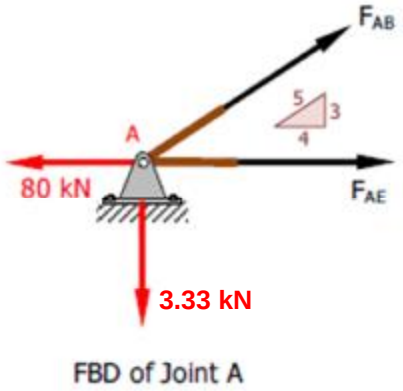
$\Sigma F_y = \frac{3}{5} F_{AB} - 3.33 = 0$

$F_{AB} = 5.56$ kN (T) ***Ans.***

$\Sigma F_x = F_{AE} + \frac{4}{5} F_{AB} - 80 = 0$

$F_{AE} + \frac{4}{5}(5.56) - 80 = 0$

$F_{AE} = 75.56$ kN (T) ***Ans.***




FBD of Joint A

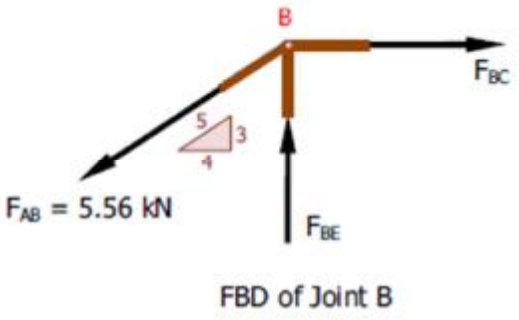
**At joint B**

$\Sigma F_x = F_{BC} - \frac{4}{5} F_{AB} = F_{BC} - \frac{4}{5}(5.56) = 0$

$F_{BC} = 4.45$ kN (T) ***Ans.***

$\Sigma F_y = F_{BE} - \frac{3}{5} F_{AB} = F_{BE} - \frac{3}{5}(5.56) = 0$

$F_{BE} = 3.34$ kN (C) ***Ans.***



FBD of Joint B

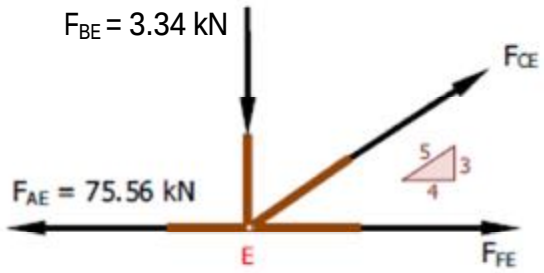
**At joint E**

$\Sigma F_y = \frac{3}{5} F_{CE} - F_{BE} = \frac{3}{5} F_{CE} - 3.34 = 0$

$F_{CE} = 5.57$ kN (T) ***Ans.***

$\Sigma F_x = F_{FE} + \frac{4}{5} F_{CE} - F_{AE} = F_{FE} + \frac{4}{5}(5.57) - 75.56 = 0$

$F_{FE} = 71.11$ kN (T) ***Ans.***



FBD of Joint E

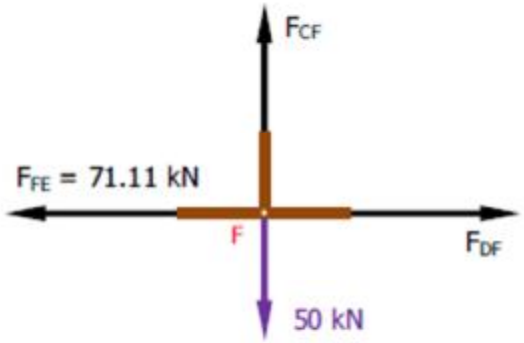
**At joint F**

$\Sigma F_y = F_{CF} - 50 = 0$

$F_{CF} = 50$ kN (T) ***Ans.***

$\Sigma F_x = F_{DF} - F_{FE} = F_{DF} - 71.11 = 0$

$F_{DF} = 71.11$ kN (T) ***Ans.***



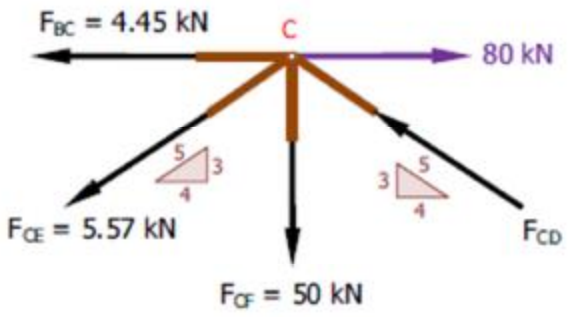
FBD of Joint F

**At joint C**

$\Sigma F_x = -\frac{4}{5} F_{CD} - \frac{4}{5} F_{CE} - F_{BC} + 80 = 0$

$-\frac{4}{5} F_{CD} - \frac{4}{5}(5.57) - 4.45 + 80 = 0$

$F_{CD} = 88.87$ kN (C) ***Ans.***



FBD of Joint C

$\Sigma F_y = \frac{3}{5} F_{CD} - \frac{3}{5} F_{CE} - F_{CF} = 0$

$\frac{3}{5}(88.87) = \frac{3}{5}(5.57) + 50$

53.3 = 53.3 ***Check***

**At joint D**

$\Sigma F_x = \frac{4}{5} F_{CD} - F_{DF} = 0$

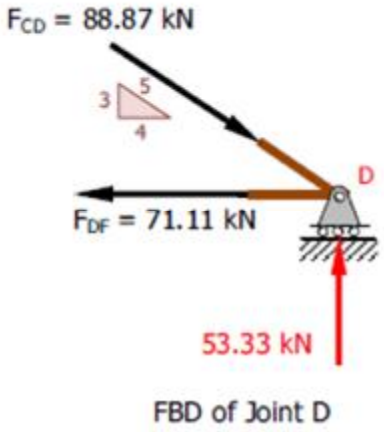
$\frac{4}{5}(88.87) = 71.11$

71.1 = 71.1 ***Check***

$\Sigma F_y = D_y - \frac{3}{5} F_{CD} = 0$

$53.33 = \frac{3}{5}(88.87)$

53.3 = 53.3 ***Check***



FBD of Joint D